CONTINUOUS MEASUREMENT OF CARDIAC OUTPUT WITH THE USE OF STOCHASTIC SYSTEM IDENTIFICATION TECHNIQUES

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ABSTRACT. The limitations of developing a technique to measure cardiac output continuously are given. Logical explanations are provided for the economic, technical, and physiologic benefits of a stochastic system identification technique for measuring cardiac output. Heat is supplied by a catheter-mounted filament driven according to a pseudorandom binary sequence. Volumetric fluid flow is derived by a cross-correlation algorithm written in the C language. In vitro validation is performed with water in a flow bench. The computed flow (y) compared with the in-line–measured flow (x) yields the linear regression y = 1.024x − 0.157 (r = 0.99). The average coefficient of variation is less than 2% over a volumetric fluid flow range of 2 to 10 L/min.

KEY WORDS: Measurement techniques: cardiac output.

THE PROBLEM

Management of critically ill patients is based on knowledge of several fundamental variables, including blood pressure, blood oxygenation, cardiac output, and temperature. Continuous measurement of these variables affords more information than do intermittent determinations, and frequently it is more accurate and involves less user frustration. Clinical continuous determination techniques are currently available for all these variables except cardiac output. A unique, clinically useful technique has been developed [1] and is described in this article.

The development of a continuous cardiac output technique is particularly difficult since several unique physiologic features render classic fluid flow techniques invalid. The most influential feature is the elastic nature of the great vessels and the tremendous variation in cross-sectional area, particularly of the pulmonary artery, that occurs with age, physical habitus, vascular volume status, body position, disease state, and drug administration. Because it is difficult to reliably estimate the cross-sectional areas of the great vessels, volumetric blood flow cannot be calculated with a measurement of blood velocity.* A second influential feature is that the distribution of blood flow from the great vessels to the peripheral arteries varies greatly under varying physiologic conditions. Determining volumetric blood flow in a peripheral artery does not allow reliable interpolation of flow in the great vessels. A further complicating fac-

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*In classic fluid mechanics, volumetric flow is the product of cross-sectional area and mean fluid velocity.

tor is that cardiac output varies with ventilation, superimposing yet another source of variability.

Numerous attempts have been made to develop techniques to measure cardiac output continuously. Velocity may be measured directly by using ultrasound transducers placed internal [2] or external [3] to the vessel. Velocity may be calculated indirectly by measuring the time of flight between two points [4] or by hot-wire anemometer methods [5]. For some applications, vessel cross section is measured [2,3] or estimated. If effort is given to measuring the geometry of the vessel cross section, flow can be calculated from blood velocity and vessel geometry [6]. Techniques to estimate the cross-sectional area of the great vessels are weakened because the vessel is not a circular cylinder and the cross-sectional area cannot be estimated reliably from a diameter. An alternate, noninvasive ultrasound approach, the attenuation-compensated volume flowmeter, uses two transducers to measure volumetric blood flow [7]. The performance of some devices is very dependent on the user's ability.

Among the various classic attempts to measure blood flow, indicator dilution techniques have gained clinical favor for intermittent determinations and have become the clinical standard for evaluation of other techniques [8]. Indicator techniques are based on conservation of mass or conservation of heat principles, measure true bulk mass movement, and are independent of the physical dimensions of the system of interest. The mathematical derivation in Appendix B demonstrates this. Some of the abbreviations and equations used in Appendix B and elsewhere in this article are defined in Appendix A.

When using dye, heat, cold, or another indicator, a known quantity of indicator is injected into the proximal vessel and its total appearance is measured at a distal point [9–12]. To develop an automated or continuous indicator-based technique to measure cardiac output, the natural extension would be to place a heating filament on a flow-directed pulmonary artery catheter. This has been done by several investigators who have used various signal-processing techniques. Khalil [13] applied indicator as a step function and measured the downstream DC change by using equation A1 from Appendix B. Normann et al [14] applied the indicator as an "impulse" and measured the area under the washout curve by using equation A2. Philip et al. [15] applied a sinusoidal input and measured downstream indicator attenuation by using a modification to equation A1. While all these techniques work, they may be severely compromised because of background thermal noise in the pulmonary artery [16–20] or limitations on maximum peak heat flux or temperatures [14].

THE PROPOSED SOLUTION

Fully understanding the physiologic, environmental, and technical limitations, the requirements for developing a reliable, inexpensive, and clinically useful technique to measure cardiac output continuously are obvious: (1) an indicator dilution technique should be used; (2) heat is the most practical indicator; (3) for a catheter-mounted filament, the maximum deliverable heat flux is temperature limited [21], yielding a relatively small signal in flowing blood†; and (4) most of all, the signal processing technique must be robust enough to perform in a noisy thermal environment.

The combination of thermal indicator dilution with stochastic system identification techniques satisfies these requirements. Stochastic techniques differ from classical deterministic techniques in that the statistical properties of the input and output signals are of more interest than the instantaneous values of the signals themselves. Many excellent books have been written over the decades on stochastic techniques [22–26]. Appendix C gives a summary of the stochastic theory underlying the proposed method.

Because the convolution equation (Equation C1) is difficult to invert, some simplifying techniques must be used. Note that the sum is simplified if the autocorrelation, $\phi_{xx}(\overline{i - k\Delta t})$, is nonzero only when $i = k$ (white noise). Then, the equation reduces to $\phi_{xy}(k\Delta t) = C\ h(k\Delta t)$, where C is a constant. A true white noise input is impossible to realize, so much effort has been devoted to developing input waveforms that yield an autocorrelation similar to that of white noise. Although there are numerous such waveforms [27], one particular class, known as pseudorandom binary codes, has particularly desirable characteristics. Appendix D gives a brief description.

Grasping a stochastic approach to measurement of cardiac output requires an understanding of several additional key principles. Stochastic system identification, as exemplified in equation C1, is applied commonly to measure a system's impulse response. It must be realized that part of the indicator dilution technique involves measuring the vascular impulse response, for the "washout" curve is the scaled impulse response. In addition, the system under investigation should be approximately linear and time invariant over the measurement interval. Excellent research has been conducted in documenting that vascular beds fulfill these requirements [28–34].

†The catheter filament temperature is a function of filament power, catheter surface area, and blood velocity.

To measure cardiac output, one must combine the stochastic system identification techniques with the equations governing conservation of heat [1]. A rigorous derivation is given in Appendix E. Equation E6 is used to calculate flow:

$$F = \frac{P(60)/4{,}180}{\left(\frac{2N}{N+1}\right) \rho c \sum_{k=0}^{N-1} \phi_{a\Delta T}(k\Delta t)}, \qquad \text{(E6)}$$

where $\Sigma_{k=0}^{N-1} \phi_{a\Delta T}(k\Delta t)$ is the area under the input-output cross-correlation as explained in Appendix E and P is the heater power in the *on* state.

Figure 1 is a block diagram that demonstrates the procedure used in a physiologic preparation. A flow-directed pulmonary artery catheter is positioned such that the heating filament is in the right ventricle and the distal thermistor in the pulmonary artery. A clock (1) is used to generate the pseudorandom binary sequence with a clock state duration of Δt (here set to 1 second) and a code of length fifteen. The binary code generator (2) turns the analog filament driver (3) on or off, causing the filament to deliver either no heat or approximately 15 W of heat. The analog thermistor reader (4) measures the pulmonary temperature. Both the binary code and the corresponding measured temperature are stored in ensemble buffers (5a, 5b). Once sufficient ensembles have been collected, usually approximately ten, the cross-correlation is computed. The cross-correlator (6) yields the indicator washout curve from which the flow is computed (7) by using equation E6.

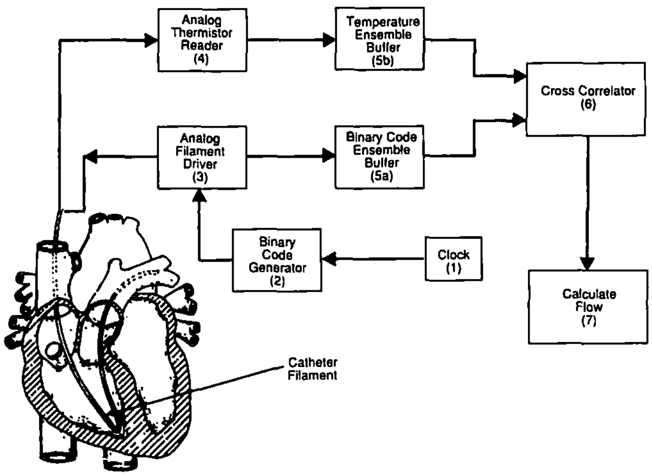

Fig 1. A block diagram of a catheter-mounted filament driven according to a pseudorandom binary sequence. Cardiac output is calculated by a cross-correlation algorithm. (See text for elaboration.)

The Algorithm

The heart of the algorithm is the cross-correlation. The following program, written in C language, is used.

```
/*****************BEGIN************/
/*Cross-correlation Algorithm
**Mark Yelderman
*/
#include <stdio.h>
#include <math.h>

main()
{
int icod[15];                 /*pseudorandom binary code*/
float cross_cor[15];          /* cross-correlation function*/
float temp_buf[150];          /*stores detected temperature */
float x_in,y_out;             /*system input and output*/
float area,flow;
float pow;                    /*pow is filament power*/
float dt;                     /*dt is the real-time state duration*/
int index,idat;
int i,j,k;                    /* loop counters*/
int irun;                     /*irun is number of runs of code*/
/*Subroutines are listed next*/
float sys_IO();               /*the general I/O subroutine*/
float wait();                 /*system real-time clock call*/
/* ********************************************** */
```

```
/*The first step is to select a binary code. */

/*a code of length 15 will be used. */
  icod(0)  =  1;
  icod(1)  = -1;
  icod(2)  =  1;
  icod(3)  =  1;
  icod(4)  =  1;
  icod(5)  =  1;
  icod(6)  = -1;
  icod(7)  = -1;
  icod(8)  = -1;
  icod(9)  =  1;
  icod(10) = -1;
  icod(11) = -1;
  icod(12) =  1;
  icod(13) =  1;
  icod(14) = -1;

  pow = 15.0;                          /* set catheter filament power in watts*/
  dt = 1.0;                            /*set real-time clock duration in seconds*/
/*The system must be 'loaded' with one run before starting*/
for (i = 0; i <= 14; i++)
  {
  if(icod[i]==1) x_in = pow;          /*if the code is 1, filament is turned on*/
    else x_in = 0.;                    /*if the code is 0, filament is turned off*/
  y_out = sys_IO(x_in);               /*for this state, the system I/O is accessed*/
  wait(dt);                            /*wait for next clock*/.
  cross_cor(i) = 0.;                   /*zero for next loop*/
  }
/*now start the actual test*/
irun = 8;                              /* this may be set to any value less than 10
                                       ** otherwise temp-buf will overflow*/
idat = 0;                              /*idat is the buffer index */
for (j = 0; j <= irun-1; j++)
{
for (i = 0; i <= 14; i++)
  {
  if(icod[i]==1) x_in = pow;
    else x_in=0.;
  y_out = sys_IO(x_in);
  temp_buf[idat++] = y_out;           /*detected temperature stored in temp_buf*/
  wait(dt);                            /*wait for next*/
  }
}

/* The cross-correlation is performed for the irun runs*/
idat = 0;                              /*reinitialize the buffer index counter*/
for (j = 0; j <= irun - 1; j++)
{
for (i = 0; i <= 14; i++)
  {
  for (k = 0; k <= 14; k++)
  {
  index = i - k;                       /* index is the offset for the cross-correlation
  if(index < 0) index += 15;           /* Because the code repeats, the */
                                       /*same code is used over*/
  if(icod[index] == 1) cross_cor[k] += temp_buf[idat++];
                  else cross_cor[k] -= temp_buf[idat++];

  }
  }
}
/*scale cross-correlation; see equation E6*/
for (i = 0;i <= 14;i++)cross_cor[i] /= (irun*15)*2*15/16;
```

```
/* calculate cardiac output or flow*/
area = 0.;
for (i = 0; i <= 14;i++)
   area+ =cross_cor[i];
flow = (pow/4180.)*60./(.87*1.05*area);          /*See equation E6*/
return(0);
}
/*******************************/
/* system I/O subroutine follows */
/*this is a general subroutine which interfaces to the
     system being tested*/
float sys_IO(x_in)
float x_in, y_out;
int err;
{
err=fil_driver(x_in);                            /*call to filament driver*/
y_out= temp_driver;                              /*call to temperature driver*/
return(y_out);
}
/*******************************/
/*system wait subroutine follows*/
float wait (dt)
float dt;
{
/* use system real time clock calls*/
}
return(0);
/****************END************/
```

Constraints

The use of stochastic processes for system identification is complex and requires the researchers to select and apply numerous constraints appropriately. The length of the pseudorandom binary code is important. A shorter code provides greater signal strength at each frequency, but provides less resolution in the final cross-correlation. For this application, the best selection is a code of length fifteen. Regardless of the code length, the duration of the code run must be greater than the time of a washout curve; in addition, at least one run must be made prior to the initialization of the cross-correlation to "load" the system. The number of runs used in the cross-correlation is a function of the signal-to-noise ratio. In the absence of noise, the cross-correlation converges in one run, but as the thermal noise of the system increases, more runs are required. This reflects the combined effects of signal power, system thermal noise, and the observation interval or run time in influencing the accuracy of the final calculation. An ensemble length of ten runs in this application provides a final result in 5 minutes with an error of less than 1%.

The analog-to-digital (A/D) conversion rate must exceed the pseudorandom binary state rate and be at least twice as fast as any noise components. Using an appropriate low-pass filter before the A/D converter and sampling at five to ten times the pseudorandom binary state rate is appropriate. With such a high sample rate, the cross-correlation may be done by first averaging the sampled values for each binary state and then cross-correlating, or cross-correlating the entire observation buffer, that is, pseudorandom binary code times the number of runs times the samples of each code state. The C-language algorithm uses only one sample per pseudorandom binary state for simplicity. The cross-correlation in Figure 2 is obtained by cross-correlating the entire observation buffer, which contains ten samples per code state.

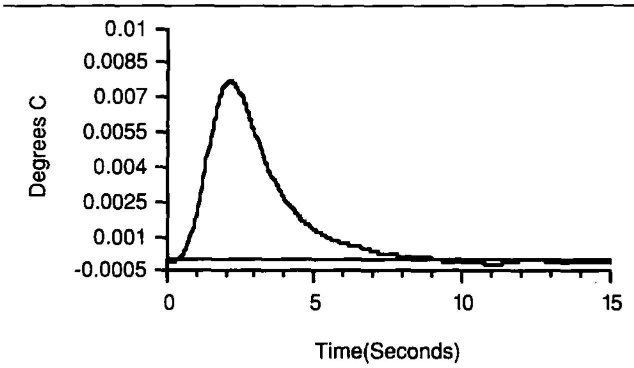

Fig 2. The cross-correlation of the two traces of Figures 3 and 4.

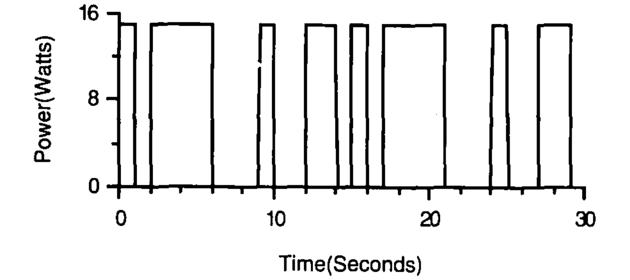

Fig 3. Input power. Two cycles of a pseudorandom binary code of length fifteen.

Evaluation of Performance

The method can be applied to any indicator dilution technique and can be evaluated in a bench model. For our purposes, at constant temperature, a constant flow of water is directed through a test chamber. The test chamber has a volume comparable to that of a heart and a volumetric flow rate that is adjustable from 2 to 10 L/min. A thermal transducer or heating filament is placed at the entrance to the chamber and a thermistor is placed at the exit of the chamber. The true flow rate is measured by an in-line electronic flowmeter with a documented accuracy and stability error less than 1% over the range of flows of interest. Careful attention is given to analog circuitry, namely, amplifier gain, heater resistance, and thermistor gain. A sample of two filament power runs of length fifteen is shown in Figure 3, and the resultant temperature profile seen at the thermistor is shown in Figure 4.

These two curves are cross-correlated according to the classic equation derived in Appendix E, namely,

$$\frac{1}{N}\sum_{i=0}^{N-1} \Delta T(i\Delta t)\ \overline{a(i + k\Delta t)} = \phi_{a\Delta T}(k\Delta t).$$

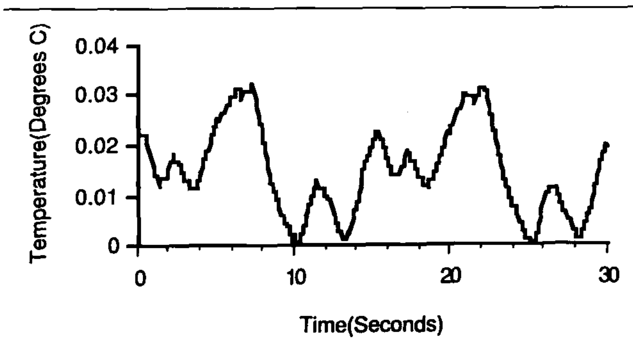

Fig 4. Output temperature. The distal temperature detected as a result of a proximal input of 15 W of peak power applied according to the pseudorandom code of Figure 3.

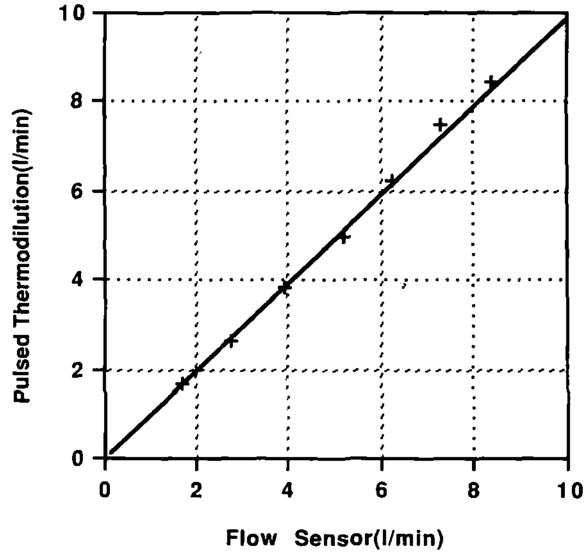

Fig 5. Linear regression of data obtained in a water model. True flow (abscissa) is measured with an in-line flow transducer and the compared flow (ordinate) is determined by using the algorithm in the text.

The resulting cross-correlation, $\phi_{a\Delta T}(k\Delta t)$, is the usual indicator dilution curve (see Fig 2). The area under the indicator dilution washout, cross-correlation function $\phi_{a\Delta T}(k\Delta t)$, is computed and the flow equation evaluated.

The system was evaluated with a peak thermal transducer power (P) of 15 W and a total observation interval of ten runs of the pseudorandom binary code. Linear regression of the data (Fig 5) yielded $y = 1.024x - 0.157$ ($r = 0.99$). The standard error of the estimate of y on x (S_{xy}) is 0.13. The average coefficient of variation is less than 2%.

DISCUSSION

The technique of measuring volumetric fluid flow is derived from both conservation of mass and stochastic system identification principles. The technique is straightforward to implement, measures true volumetric flow, requires no user calibration, and is independent of the physical geometry of the system. The required devices can be mounted on a pulmonary artery catheter and used clinically to measure cardiac output continuously and with minimal user effort.

The stochastic system identification principles provide two other advantages. Because the pseudorandom sequence involves a duty cycle of approximately 50%, heat is supplied to the fluid without requiring high peak power or temperature levels. The actual filament fluid boundary temperature is a function of power heat density, filament surface area, and fluid velocity; however, for this configuration, boundary surface temperatures are approximately 5°C above ambient fluid temperature at 6 L/min.

This stochastic system identification technique was evaluated in a water bench model, which has little thermal background noise and is found to be accurate and precise over a wide range of flows. When the technique is evaluated in in vivo preparations, a greater challenge is present because of the thermal background noise. The ability of this technique to function in a thermally noisy environment has been addressed [35].

APPENDIX A

Some of the abbreviations and equations used in this article are defined here.

m	mass (g)
ρ	density (g/ml) (a value of 1.05 for blood [18], a value of 1.00 for water)
ΔT	change in temperature (°C)
F	volumetric flow (ml/s or L/min)
Q	heat infusion (calories)
q	heat bolus (calories)
Heat capacity	heat (cal)/temperature change (°C) = $\Delta Q/\Delta T$
c	specific heat = heat capacity/mass = $\Delta Q(\text{cal})/\Delta T m(°\text{C-g})$; (for blood, a specific heat (c) of 0.87 is acceptable [18]).
x	input (flow) to the two-port system
y	output (flow) from the two-port system
h	system impulse response
$\bar{a}$	pseudorandom binary code with states of 1, 0
a	pseudorandom binary code with states of 1, -1
$\phi_{xx}(k\Delta t)$	$\frac{1}{N}\sum_{i=0}^{N-1} x(i\Delta t)\, x(\overline{i + k\Delta t})$ (autocorrelation)
$\phi_{xy}(k\Delta t)$	$\frac{1}{N}\sum_{i=0}^{N-1} x(i\Delta t)\, y(\overline{i + k\Delta t})$ (cross-correlation); $\phi_{xy}(k\Delta t)$ is the cross-correlation between the input binary code (x) and the system output (y)
Δt	duration of a state (seconds)
N	period of pseudorandom binary sequence
P	peak filament power (watts)

APPENDIX B

The thermal dilution equation to calculate average flow is based on the conservation of heat equation. Known as the first law of thermodynamics [36,37], total energy entering a control volume must be equal to the sum of the increase of energy in the control volume and the energy leaving the control volume. Assuming that indicator mixing is both instantaneous and complete and that at steady state indicator does not accumulate in the mixing chamber, the basic indicator dilution equation, stated in terms of differentials, is $dI(t) = c(t) \times F(t) \times dt$, where the various quantities have the following meanings and units: Indicator amount = (indicator amount/ml blood) × (ml blood/s) × (second).

When heat is the indicator, dI(t) is written dQ(t), and the indicator concentration function, c(t) (in cal/ml blood), is defined as $c(t) = \rho\, c\Delta T(t)$ [37], where ρ is the density of blood (in g/ml), c is the specific heat of blood (in cal/°C · g), and $\Delta T(t)$ is the associated temperature rise (in °C) as a function of time t. Substitution of the appropriate expressions for dI(t) and c(t) yields the differential form of the thermal dilution equation*:

$$dQ(t) = \rho\, c\Delta T(t)F(t)dt. \tag{A1}$$

If a bolus of heat is introduced following time zero, equation A1 may be integrated to obtain the following:

$$\int_0^\infty dQ(t) = \rho\, c \int_0^\infty \Delta T(t)F(t)dt. \tag{A2}$$

If the flow rate is a constant F and if the total heat in the bolus is $q = \int_0^\infty dQ(t)$, then the foregoing may be solved for the constant flow, yielding the thermal dilution equation for bolus injection [39–41]:

$$F = \frac{q}{\rho\, c \int_0^\infty \Delta T(t)dt}. \tag{A3}$$

If, instead of using a heat bolus, heat is pumped into the system at a varying rate, $(dQ/dt) = \dot{Q}(t)$, then equation A1 becomes

$$\dot{Q}(t) = \rho\, c\Delta T(t)F(t). \tag{A4}$$

*Mixing is not instantaneous and can be incomplete. At certain low flow rates and rapidly changing heat infusion rates, the mixing chamber capacitance may have an effect. The heat balance equation becomes

$$\frac{dQ(t)}{dt} = \rho c F(t)\Delta T(t) + \rho c V \frac{d\Delta T(t)}{dt},$$

where V is the effective mixing volume. A more detailed discussion is given by Rubin [38] and Philip et al [15].

This form of the thermal dilution equation is central to the present method and is basic to the development of the fundamental equation in Appendix E.

Equation A4, in its simple form, can be used to measure flow. If a constant heat infusion is used, call it $\dot{Q}(t) = \dot{Q}$, and the flow rate is a constant F, then solving equation A4 for the flow yields the thermal dilution equation for constant heat infusion:

$$F = \frac{\dot{Q}}{\rho\, c\Delta T}. \tag{A5}$$

Since heat infusion rate and flow rate are constant, the temperature rise is also a constant. This method has been used successfully in laboratory bench and animal studies [13,42,43].

APPENDIX C

Stochastic Signal Processing Techniques

Engineers are familiar with the discrete system impulse response or convolution sum,

$$y(k\Delta t) = \sum_{i=0}^{N-1} h(i\Delta t)\, x(\overline{i - k\Delta t}),$$

where $x(k\Delta t)$ is the input to the system, $y(k\Delta t)$ is the output, and $h(k\Delta t)$ is the characteristic system impulse function. Because the convolution equation is difficult to invert reliably in practice, numerous techniques are used to measure the system impulse response. For example, if the classic unit impulse is used, $x(\overline{i - k\Delta t})$ has a value only when $i = k$, and the output is the system impulse response $y(k\Delta t) = h(k\Delta t)$.

An alternative to using deterministic signals is to use random signals and determine the properties of the input and output signals of a system. For example, the input to the system could be a random process. If the input is $x(i\Delta t)$, then it is useful to define the autocorrelation of the input as follows:

$$\phi_{xx}(k\Delta t) = \frac{1}{N}\sum_{i=0}^{N-1} x(i\Delta t)\, x(\overline{i + k\Delta t}),$$

where $x(i\Delta t)$ is the input, $x(\overline{i + k\Delta t})$ is a delayed version of the input, and N is the observation interval.

The method of evaluating this function is straightforward. The function $x(i\Delta t)$ is duplicated but shifted by an amount k. Then, for each value of k desired, the two functions of $x(i\Delta t)$ are multiplied together for each time and the products averaged.

In a similar fashion, it is useful to define the relationship between the input and the output of a system. This relationship, known as the cross-correlation, is defined as follows:

$$\phi_{xy}(k\Delta t) = \frac{1}{N}\sum_{i=0}^{N-1} x(i\Delta t)\, y(\overline{i + k\Delta t}),$$

where $y(\overline{i + k\Delta t})$ is a delayed version of the output.

Evaluation of this function is straightforward. The function $y(i\Delta t)$ is shifted by an amount k. Then, for each value of k desired, the shifted function $y(\overline{i + k\Delta t})$ is multiplied by $x(i\Delta t)$ for each time and the products averaged.

Once these relationships are developed, the system impulse response is defined by a different convolution equation, namely:

$$\phi_{xy}(k\Delta t) = \sum_{i=0}^{N-1} h(i\Delta t)\, \phi_{xx}(\overline{i - k\Delta t}). \tag{C1}$$

This equation implies that one need only observe the input and output of a system regardless of what they are,† until the autocorrelation and cross-correlation converge. Then, the system impulse response can be determined by the convolution summation.

Because this equation is difficult to solve, numerous simplifying techniques are used. The most obvious one is to use broad-band white noise as the input. Because white noise is completely uncorrelated with itself, its autocorrelation is nonzero at only one point, namely, $i = k$. In this case, the cross-correlation is proportional to the system impulse response: $\phi_{xy}(k\Delta t) = C\, h(k\Delta t)$. C a constant.

APPENDIX D

Pseudorandom Binary Codes

A full development of pseudorandom binary codes is beyond our scope and is available elsewhere [27]. Pseudorandom binary codes are so named because, even though they are binary sequences with deterministic properties and are periodic, the autocorrelation of such codes closely resembles that of random or white noise processes. Several essential properties are of interest. The pseudorandom binary codes can be physically realized with *n*-stage shift registers with appropriate feedback loops. Each stage has a unique binary state, for example, -1 and 1, and the periodicity of the last stage (the output) can be equal to $N = 2^n - 1$, where n is the number of stages. This results in codes of total length N = 7, 15, 31, 63, etc. Two periods of a sample code of length seven are shown in Figure 6. The duration of each state (Δt) is arbitrary and in this example equals 1 second. The only limitation is that the total period, (ΔtN), must be longer than the total impulse response time of the system being measured, h(k). Also note that there is one more high than low state.

The Fourier series of the binary code reveals the frequency content. The series involves N frequencies, $(1/\Delta tN)$, $(2/\Delta tN)$, . . . , $(N/\Delta TN)$, each with an amplitude, $[\sin(2\pi j/\Delta tN)]/(2\pi j/\Delta tN)$. A normalized power-density spectrum is shown in Figure 7.

Our interest is in the autocorrelation properties of the pseudorandom binary sequence. As an example, consider a code of length seven, namely, -1, 1, 1, 1, -1, 1, -1. The autocorrelation is given by the discrete equation:

$$\phi_{xx}(k\Delta t) = \frac{1}{N}\sum_{i=0}^{N-1} x(i)\, x(\overline{i + k\Delta t}).$$

†There are some limitations, the main one being that the bandwidth of the input signal must be broader than the bandwidth of the system response.

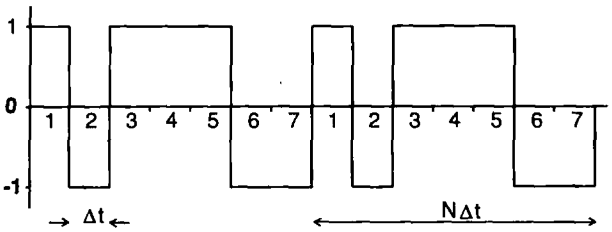

Fig 6. Two runs of a pseudorandom binary code of length seven. Δt = *state_duration (seconds);* N = *period of pseudorandom binary sequence.*

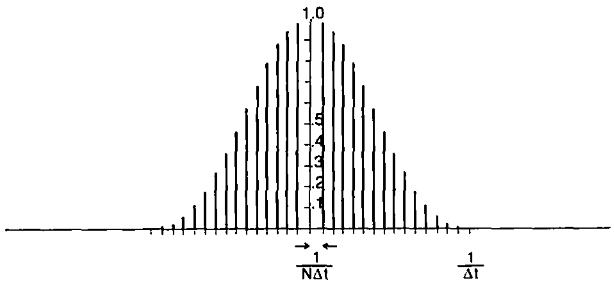

Fig 7. The frequency content of a pseudorandom binary code of length fifteen. N = *period of pseudorandom binary sequence;* Δt = *state duration (seconds).*

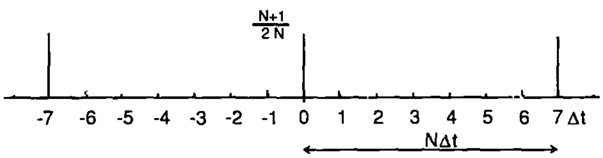

Fig 8. The cross-correlation of pseudorandom binary codes a (1, −1) and ā (1, 0) of length seven. N = *period of pseudorandom binary sequence;* Δt = *state duration (seconds).*

For the case of a delay equal to zero, k = 0, the code is −1, +1, +1, +1, −1, +1, −1. Then, from the definition, the zero-lag autocorrelation is calculated as

$$\phi_{xx}(0) = \frac{1}{7}\left[\begin{array}{l}(-1)\cdot(-1) + (1)\cdot(1) + (1)\cdot(1) + (1)\cdot(1) \\ + (-1)\cdot(-1) + (1)\cdot(1) + (-1)\cdot(-1)\end{array}\right]$$

$$= 1.$$

For the case of a delay equal to one, k = 1, the unshifted code is multiplied by the code shifted by one time interval. The unshifted code remains − 1, + 1, + 1, + 1, − 1, + 1, − 1. The one-shifted code is − 1, − 1, + 1, + 1, + 1, − 1, + 1. Hence, the one-lag autocorrelation is

$$\phi_{xx}(1) = \frac{1}{7}\left[\begin{array}{l}(-1)\cdot(-1) + (1)\cdot(-1) + (1)\cdot(1) + (1)\cdot(1) \\ + (-1)\cdot(1) + (1)\cdot(-1) + (-1)\cdot(1)\end{array}\right]$$

$$= -\frac{1}{7}.$$

The process is continued for other values of k, both positive and negative. It can be demonstrated for the general case that, for a code having amplitudes ±1:

$$\phi_{xx}(k\Delta t) = +1 \text{ for } k = 0, N, 2N, \ldots$$

$$= -\frac{1}{N} \text{ otherwise.}$$

For our purposes, the input sequence used has the two states 1 or 0. However, in Appendix E, cross-correlation is performed by using a code with states 1 or −1. The cross-correlation between a and ā has these values [27]:

$$\phi_{a\bar{a}}(k\Delta t) = \frac{N+1}{2N} \text{ for } k = 0, N, 2N, \ldots \quad \text{(D1)}$$

$$= 0 \text{ otherwise.}$$

This cross-correlation, $\phi_{a\bar{a}}(k\Delta t)$, is shown in Figure 8. Note that it has a nonzero value only at k = 0, N, 2N, . . . and is central to the calculation in Appendix E.

APPENDIX E

Conservation of Heat and Stochastic Processes

When a stochastic process is used to determine the system impulse response, it is necessary to derive the conservation of heat equation that will allow the calculation of volumetric flow. Recall the basic heat infusion equation (A4):

$$\dot{Q}(t) = \rho c \Delta T(t) F(t). \quad \text{(A4)}$$

This can be written in the discrete form:

$$\dot{Q}(i\Delta t) = \rho c \Delta T(i\Delta t) F(i\Delta t). \quad \text{(E1)}$$

Assuming the flow is a constant F and thermal power is supplied with amplitude $P\bar{a}(i\Delta t)$, where P is in watts and $\bar{a}(i\Delta t)$ is the pseudorandom binary code of amplitude 1 or 0, then, making the relevant substitutions (and converting from cal/s to watts) yields

$$P\bar{a}(i\Delta t) = 4.18 \rho c \Delta T(i\Delta t) F. \quad \text{(E2)}$$

Multiplying both sides of equation E2 by $1/Na(\overline{i + k\Delta t})$ and summing on i from zero to N − 1 yields

$$P\frac{1}{N}\sum_{i=0}^{N-1} \bar{a}(i\Delta t)a(\overline{i + k\Delta t}) \quad \text{(E3)}$$

$$= 4.18 \rho c F \left\{\frac{1}{N}\sum_{i=0}^{N-1} \Delta T(i\Delta t)a(\overline{i + k\Delta t})\right\}.$$

Now, the lefthand side is just $P\phi_{a\bar{a}}(k\Delta t)$. The brace on the righthand side is $\phi_{a\Delta T}(k\Delta t)$. Hence, equation E3 may be simplified to

$$P\phi_{a\bar{a}}(k\Delta t) = 4.18 \rho c F \phi_{a\Delta T}(k\Delta t). \quad \text{(E4)}$$

Both sides of equation E4 are summed on k. Using equation D1, namely, that $\phi_{a\bar{a}}(k\Delta t)$ equals N + 1/2N when k = 0 and is zero for $k = 1, 2, \ldots, N - 1$, equation E4 reduces to

$$P\left(\frac{N+1}{2N}\right) = 4.18\rho cF \sum_{k=0}^{N-1} \phi_{a\Delta T}(k\Delta t). \tag{E5}$$

If F is to be measured in L/min rather than ml/s, multiply F by 1,000/60 and solve for F:

$$F = \frac{P(60/4,180)}{\left(\frac{2N}{N+1}\right) \rho c \sum_{k=0}^{N-1} \phi_{a\Delta T}(k\Delta t)}. \tag{E6}$$

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