

**SIZE AND SHAPE OPTIMIZATION OF FRAME AND TRUSS STRUCTURES
THROUGH EVOLUTIONARY METHODS**

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ABSTRACT

Optimization of truss and frame structures is a popular topic in mechanical, civil, and structural engineering due to the complexity of problems and benefits to industry. Current structural optimization software packages often lack the ability to find original optimal designs because of their *deterministic* nature, while those employing *stochastic* methods are not tailored specifically for frame and truss structures. A customized genetic algorithm was developed to aid in the structural design process.

The basis of this optimization software was a standard generational genetic algorithm combined with parsimony pressure and allele copying. These features, in addition to others, were tested and adjusted to solve a wide variety of structural optimization problems. The result is a usable software package with many flexible features that give the user complete control of the design process. The interface has been kept concise in order to maintain speed of execution and ease of use.

Three design problems were explored using the software: a 10-bar truss benchmark, a 25-bar truss benchmark, and a miniature race car space frame. The truss structures verify the accuracy of calculation and prove the superior nature of the search algorithm. The race car frame shows feasibility of application to a real engineering problem.

The genetic algorithm presented is a robust structural optimization tool that may be used with varying amounts of interaction. It is tailored for most structural optimization problems to produce original solution sets not found using commercial software packages.

ACKNOWLEDGEMENTS

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NOMENCLATURE

STRUCTURAL ANALYSIS

| <i>Symbol</i> | <i>Description</i> | <i>Units</i> |
|---------------|---|---------------------|
| E | Modulus of Elasticity | force/area |
| ν | Poisson's Ratio | -- |
| s_y | Yield Strength | force/area |
| g | Unit Weight | force/volume |
| ρ | Mass Density | mass/volume |
| A | Cross Sectional Area | area |
| I_y | Moment of Inertia about the Weak Axis | length ⁴ |
| I_z | Moment of Inertia about the Strong Axis | length ⁴ |
| J | Torsional Constant | length ⁴ |
| Z_y | Section Modulus for the Weak Axis | length ³ |
| Z_z | Section Modulus for the Strong Axis | length ³ |
| $MASS$ | Structural Mass/Weight | mass/weight |
| K | Stiffness Matrix | force/length |
| F | Load Vector | force |
| Y | Intermediate Vector | length |
| U | Displacement Vector | length |

GENETIC ALGORITHM

| <i>Symbol</i> | <i>Description</i> | <i>Units</i> |
|-----------------------------------|---|--------------|
| <i>INDEX</i> | Selection Index | -- |
| <i>POPSIZE</i> | Population Size | -- |
| <i>RANDOM</i> | Pseudorandom Number [0,1) | -- |
| <i>BIAS</i> | Selection Bias [0,1) | -- |
| <i>MB</i> | Mutation Bias [0,1) | -- |
| <i>FITNESS</i> | Structural Fitness Value | -- |
| <i>dW</i> | Displacement Weighting Constant | -- |
| <i>sW</i> | Stress Weighting Constant | -- |
| <i>dV</i> | Displacement Violations | -- |
| <i>sV</i> | Stress Violations | -- |
| <i>l</i> | Loading Case | -- |
| <i>n</i> | Node Number | -- |
| <i>e</i> | Element Number | -- |
| <i>NL</i> | Number of Loading Cases | -- |
| <i>NN</i> | Number of Nodes | -- |
| <i>NE</i> | Number of Elements | -- |
| <i>D_{l,n}</i> | Displacement Violation Magnitude at Load Case <i>l</i> and Node Number <i>n</i> | -- |
| <i>S_{l,e}</i> | Stress Violation Magnitude at Load Case <i>l</i> and Node Number <i>n</i> | -- |
| <i>displacement_{l,n}</i> | Nodal Displacement at Load Case <i>l</i> and length Node Number <i>n</i> | |
| <i>stress_{l,e}</i> | Element Stress at Load Case <i>l</i> and Element Number <i>e</i> | force/area |
| <i>dLIMIT_n</i> | Displacement Limit at Node Number <i>n</i> | length |
| <i>sLIMIT_e</i> | Stress Limit at Element Number <i>e</i> | force/area |

GLOSSARY

| <i>Term</i> | <i>Definition</i> |
|--------------------------|--|
| Allele | An object or value located at a <i>locus</i> . |
| Child | The resultant <i>individual</i> from the <i>recombination</i> of <i>parents</i> . |
| Chromosome | A collection of <i>loci</i> making up a point in the space where the evolutionary search takes place. |
| Constraint | A boundary placed on an <i>allele</i> used in the <i>genotypic</i> space. |
| Convergence | The point at which little or no progress is made. |
| Deterministic | Results are completely determined by an initial state. |
| Element | An edge defined by two <i>nodes</i> that contains material and cross-sectional properties. |
| Elitist | An <i>individual</i> guaranteed to participate in the next <i>generation</i> without <i>recombination</i> or <i>variation</i> based on fitness. |
| Exploitation | The concentration of search in a known region of the search space. |
| Exploration | Search method that tests unexplored regions of the search space. |
| Generational GA | <i>Population</i> model in which one generation of <i>individuals</i> creates the next generation through <i>selection</i> , <i>recombination</i> , and <i>variation</i> . After each generation the whole <i>population</i> is replaced by the <i>children</i> created. |
| Genetic Algorithm | An optimization algorithm that typically has a binary representation, a population selection, and an emphasis on genetically inspired recombination. |
| Genotype | Internal abstract representation used for <i>chromosomes</i> . |
| Heredity | Concept that genetic information is passed from <i>parent</i> to <i>child</i> through the generations. |
| Individual | A <i>chromosome</i> representing a single solution. |
| Load | A force or moment applied at a <i>node</i> . |

| | |
|------------------------------|---|
| Locus | A single position or placeholder in the <i>chromosome</i> . |
| n-Parent Crossover | Any number (n) of <i>parents recombining</i> to produce <i>children</i> . |
| Node | A vertex located in three-dimensional space. |
| Parent | An <i>individual</i> that has been <i>selected</i> to perform <i>recombination</i> or <i>variation</i> . |
| Parsimony Pressure | Applying pressure to a <i>population</i> by introducing increasing penalties to the fitness function for solution complexity. |
| Phenotype | A physical representation of a solution to the problem. |
| Population | A multiset of <i>chromosomes</i> . |
| Problem Solver | The engineer or scientist conducting design through the software. |
| Recombination | The act of exchanging information in the representation between <i>parents</i> to create <i>children</i> . |
| Selection | Mechanism for employing pressure to a <i>population</i> based on quality. |
| Shape Optimization | The act of changing <i>node</i> locations while maintaining <i>topology</i> in order to produce optimal structures. |
| Size Optimization | The act of changing <i>element</i> cross-sectional properties in order to produce optimal structures. |
| Stochastic | Results depend on the outcomes of random choices. |
| Structure | A set of <i>nodes</i> that are connected by a set of <i>elements</i> with applied <i>loads</i> and <i>constraints</i> . |
| Topology Optimization | The act of changing <i>node</i> connectivity in order to produce optimal structures. |
| Variation | Mechanism for introducing new genetic material into the <i>population</i> through small changes in a <i>chromosome</i> . |
| Verbose | Communication from computer software to the user through text messages on the display. |

1.0 INTRODUCTION

Structural design is a branch of Engineering that deals with systems comprised from a set of structural members. These members may be characterized as either truss or frame elements, connected by pinned or fixed joints. Common structures include truss bridges, frame buildings, race car and airplane space frames, crane arms, and power line truss towers. Structures may range in size from 1,450 ft. tall buildings weighing 222,500 tons down to bicycle frames weighing less than 10 lb.

Structural optimization has become a valuable tool for engineers and designers in recent years. Although it has been applied for over forty years, optimization in engineering has not been a commonly used design tool until high performance computing systems were made widely available. Structures are becoming lighter, stronger, and cheaper as industry adopts higher forms of optimization. This type of problem solving and product improvement is now a crucial part of the design process in today's engineering industry.

The topic of optimization has its mathematical roots dating back to the 1670's with the introduction of differential calculus. Its primary purpose is to find the best result to a problem given a set of circumstances. It wasn't until the early 1950's that computer-based optimization launched itself into the engineering industry. This was due to the fact that the topic lends itself to numerical computation, which is the one task in which computers have superiority over humans. Programmers immediately began introducing new optimization methods such as nonlinear programming, unconstrained optimization, and multi-objective optimization. A recent addition to the family of numerical optimization methods is that of evolutionary computation. This category of optimization includes the genetic algorithm.

These forms of computation have opened many possibilities never before achievable in optimization. The first work utilizing the genetic algorithms was done by evolving state machines in the 1960's.

The genetic algorithm is the oldest and most common form of evolutionary computation. It derives its behavior from natural evolution and genetics, following Darwin's major principles of evolution. This method relies on random actions, trial and error, and survival of the fittest to evolve solutions to optimization problems. It acquires its strength from the fact that a wide variety of problems can be driven to very good solutions by recombining parts of previous good solutions. As engineers and designers search for new optimization methods, they find that the genetic algorithm can produce results never before possible.

Complex structures become difficult to optimize when variable interactions increase. Classical optimization methods can produce sub-optimal results because of these interactions. Genetic algorithms are known for handling global optimization problems when many local optima are present in a non-continuous fitness landscape. Evolutionary methods have produced superior structures that could then be reverse engineered and explained through the eyes of traditional engineering methods. These non-obvious solutions are possible because evolutionary methods are purely driven by quality, allowing them to work outside the scope of traditional solutions.

A modified generational genetic algorithm was coupled with a linear elastic matrix analysis tool to create structural optimization software that is capable of size and shape optimization of truss and frame structures. Usability is increased by including graphical

viewing utilities for structure visualization and optimization progress. The objective of the structural optimization is the minimization of mass with optional stress and displacement soft-constraints. A typical optimization problem will utilize shape and size capabilities with stress or displacement constraints applied. These problems deal with mixed continuous and discrete search spaces, which can create non-smooth and deceptive fitness landscapes. Three example problems were conducted to show the validity of the genetic algorithm and the feasibility of use on real engineering problems.

Historic 10-bar and 25-bar truss structures from the literature were optimized to prove computational accuracy and superiority. A race car frame from the University of Idaho's Formula SAE project was then optimized to prove that realistic improvements can be made on highly constrained applications using the genetic algorithm. All cases exercise the extents of the genetic algorithm's capabilities. Results show that the software developed produces unique and superior solutions by comparison to the commercial optimization software package GENESIS®.

The genetic algorithm software is a robust and user friendly engineering tool. The variety of solutions produced by this stochastic method aid designers in the decision making process of structural optimization. Incorporating this software in structure design leads to unforeseen and optimal designs.

2.0 STRUCTURAL SIZE AND SHAPE OPTIMIZATION

To better understand optimization of structures and the focus of this work, two definitions must be stated. The first definition is that of the structure, including all implications and capabilities in the static analysis of such systems. The second definition applies to that of structural optimization, more specifically the optimization of size and shape. These two primary definitions will hold true for the entirety of this research and are derived from McGuire (2000).

A *structure* is a set of *nodes* (vertices) that are connected by a set of *elements* (edges). This includes all plane (2D) and space (3D) truss and frame structures. *Loads* may be placed at nodes to exert a force or moment on the structure. *Constraints* may be placed at nodes to restrain the structure from translation or rotation caused by nodal loads. A valid structure must constrain at least all six degrees of freedom as a system, and over constraint will generally produce stiffer structures.

All elements are associated with a material defined by a minimum of two values: modulus of elasticity (E) and Poisson's ratio (ν). These values define the element's behavior under static linear elastic loading conditions. Values used only for the optimization process include the element's yield strength (s_y), and unit weight (g) or mass density (r). These values are used for stress limit comparison and structural mass, respectively.

2.1 Truss Structures

Truss elements are one dimensional in their local coordinate system and carry only axial loads due to their pin connections at nodes. This also means that a truss node is only

allowed translational degrees of freedom. A truss element needs only a cross sectional area (A) to define its geometry due to the axial load limitation, and its length is determined by the location of its end nodes. A three-dimensional truss element has two local degrees of freedom and six global degrees of freedom, with three translational degrees of freedom at each end of the element. Figure 2.1.1 shows a three-dimensional truss element with its local and global coordinate systems, degrees of freedom, and allowable forces. The black capital symbols represent global objects, while gray lower case symbols represent local objects. It can be seen that a truss element has only one local coordinate axis (x) originating from one node and extending through the length of the element. The only forces (f_1, f_2) and displacements (u_1, u_2) allowed in this local system lie in direct axial placement with the element, and the element has two degrees of freedom. The global coordinate system (X, Y, Z) that is used in the structural analysis then causes each local object to be broken into three equivalent global components. It is then shown that the three-dimensional truss element has six global degrees of freedom, with one for each global coordinate at each end of the element.

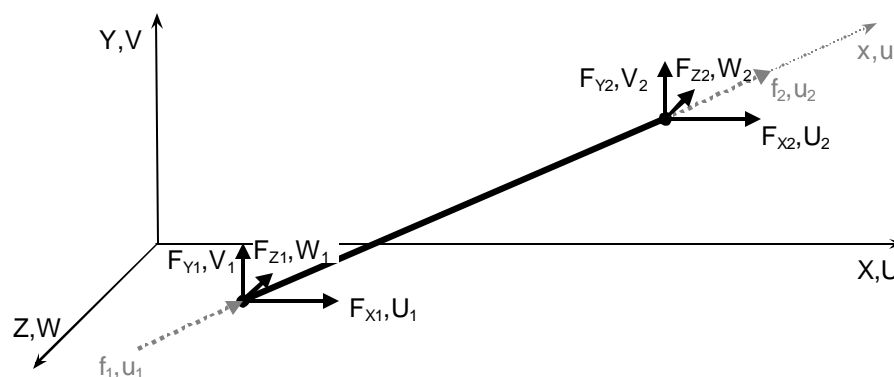


Figure 2.1.1. Truss element displaying local and global coordinate systems

2.2 Frame Structures

Frame elements carry axial, bending, and torsional loads due to their rigid connections at nodes. This means that a frame node is allowed all translational and rotational degrees of freedom. Figure 2.2.1 shows a three-dimensional frame element with its local coordinate system, degrees of freedom, and allowable forces. The transformation from local to global coordinates is analogous to that of a truss element and is not shown in the figure. It can be seen that the frame element has three local coordinate directions, allowing six forces and displacements at each end of the element. This produces a 12 degree of freedom element capable of resisting loads in any combination of directions, excluding transverse shear and bimoments (McGuire, 2000). Unlike the truss element, the number of degrees of freedom in the frame element remains constant between local and global transformations. Note that the local coordinate axes are not arbitrary in their direction and location. Following the notation used by McGuire (2000) the local x axis coincides with the centroidal axis of the element, the local y axis (weak) defines the minor principal axis of the cross section, and the local z axis (strong) defines the major principal axis of the cross section.

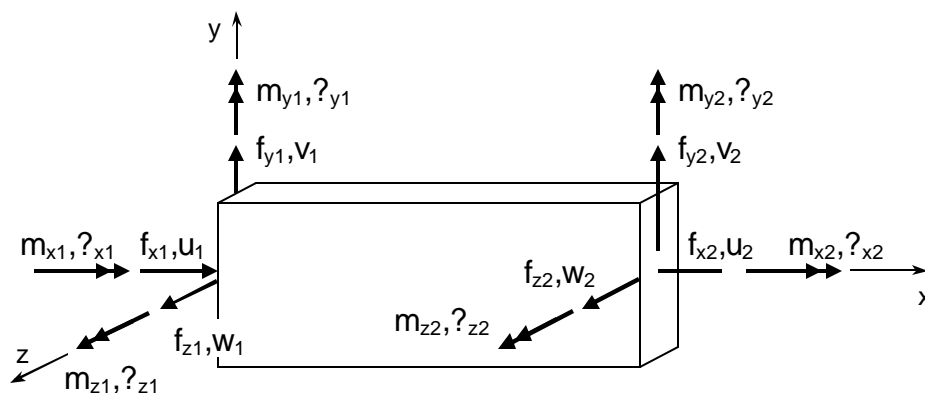


Figure 2.2.1. Frame element displaying local coordinate system

A frame element requires six dimensional properties to describe its cross section and load carrying capabilities. A cross sectional area (A), two bending moments of inertia (I_y & I_z), a torsional constant (J), and two section modulus values (Z_y & Z_z) are needed to calculate a frame element displacement and stress values due to nodal loads.

2.3 Structural Optimization Methods

Optimization of structures can be broken down to three categories: topology, size, and shape. All three categories generally have the objective of mass minimization with optional stress or displacement constraints.

- *Topology optimization* -- variance of element-node connectivity to find an optimal layout design. Difficulties may arise when a change truss topology causes the structure to become a mechanism.
- *Size optimization* -- variance of element cross sectional properties, which may be continuous or discrete variables.
- *Shape optimization* -- movement of nodes to change the shape of the structure without changing the topology. The element-node connectivity remains intact.

This work uses a combination of size and shape optimization techniques with the objective of mass minimization and the inclusion of element stress and nodal displacement constraints. Using only size and shape optimization avoids the problems associated with topological optimization while allowing substantial changes to the structure. The size and shape optimization methods are applied in parallel rather than in a series process, meaning size and shape variables are changed simultaneously.

Figure 2.3.1 shows examples of the three types of structural optimization. The topology example shows changes in connectivity between several internal elements while the node locations remain constant. The shape optimization shows a constant topology with a variation in node locations. The size optimization shows several examples of frame element cross sectional geometries that might be applied to an element. Note that truss elements may only vary their cross sectional area due to the fact that this is the only property that is needed to describe the element.

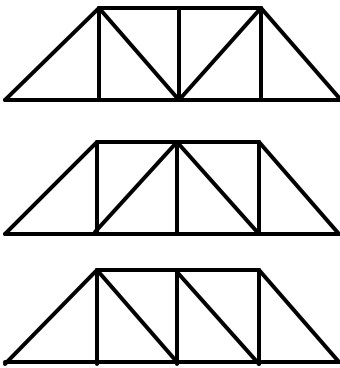
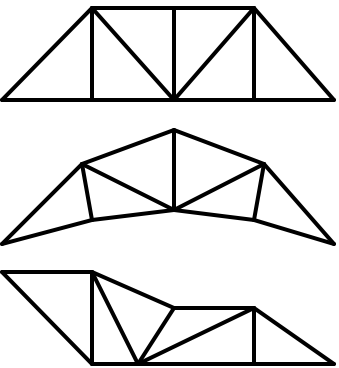
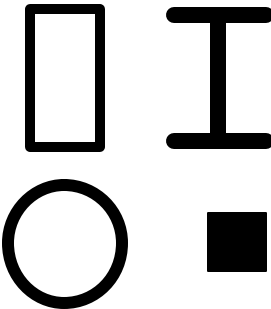
| Topology (Connectivity) | Shape (Nodes) | Size (Elements) |
|--|---|--|
|  |  |  |

Figure 2.3.1. Examples of structural optimization

Topology optimization may be applied alone or in conjunction with size and shape optimization. The combination of all three methods applied simultaneously is termed “layout” optimization. Again, this work uses only the simultaneous application of size and shape optimization approaches.

3.0 GENETIC ALGORITHM

The genetic algorithm is an area of evolutionary computation that has been in use since the 1960's. It is *stochastic* in nature, meaning that it utilizes some form of pseudorandom number generation. This, in turn, makes the solution path and results nondeterministic. Stochastic methods make use of random exploration in order to search complex landscapes riddled with local optima and deceptive solution paths. They are able to handle various types of optimization problems and can be crafted to find solutions in a reasonable amount of time. There is no guarantee of finding the global optimum or even the same local optimum between trials.

The genetic algorithm's formal definition, as defined by Eiben (2003), Goldberg (1989), and Mitchell (1998), is very general in that a designer may add features or modify existing features in order to customize the algorithm for a specific problem type. It is based on Darwin's five main principles of evolution as displayed in Figure 3.0.1: population, recombination, variation, selection, and heredity. Each principle is incorporated into the algorithm either directly or indirectly in order to mimic the natural evolutionary process. Additionally, the problem solver may include features not possibly found in nature (n-parent crossover, elitists, mutation control, etc.).

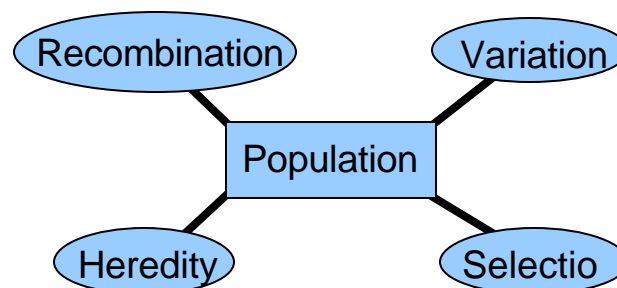


Figure 3.0.1. Darwin's evolutionary principles

3.1 Terminology and Theory

The central principal of the genetic algorithm lies in the population. Figure 3.1.1 shows a decomposition of the population model. The *population* is a collection of individuals, represented by a single box in the population. These *individuals* represent a solution to a problem, each of which should be a valid solution. An individual's genetic material is represented by a *chromosome*, which holds the genotypic form of data that can be mapped to the phenotype. The *genotype* is an abstract data set defined by the problem solver, while the *phenotype* is the physical representation describing a solution to a problem. For example, bit strings may be mapped into a decision-making process, a number or set of numbers, characters or words, etc. The chromosome is made up of *loci*, represented by the boxes in the chromosome, which are locations for values to be held. *Alleles* represent a single variable or parameter (truss area, node location in a specific direction, etc.) at a locus. In general, the population serves to retain some amount of memory about the past or present state in the optimization process. It can also be thought of as a “pool” of usable genetic information. The population is acted upon by the recombination, variation, selection, and specialty operators.

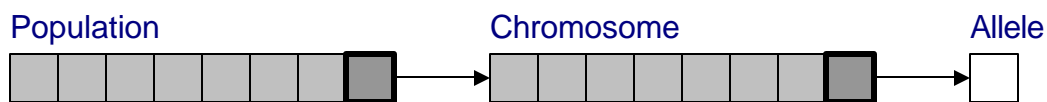


Figure 3.1.1. Visualization of population model

An example of how a structure is broken down to a chromosome representation can be seen in Figure 3.1.2. This shows a simple 5-bar/4-node truss to be optimized given the loading conditions and constraints shown. The node numbers are circled with their

coordinate locations next to them, and the element numbers are shown next to their cross sectional areas. This is a simplified 2-dimensional example, and the actual representation used in the software includes the third dimension. The node locations and cross sectional areas shown underlined represent the variables to be optimized. Notice that the second coordinate position of node 2 is not set to be optimized but is included in the chromosome representation. This is due to the software representation methods used, and is known to be left out of the optimization.

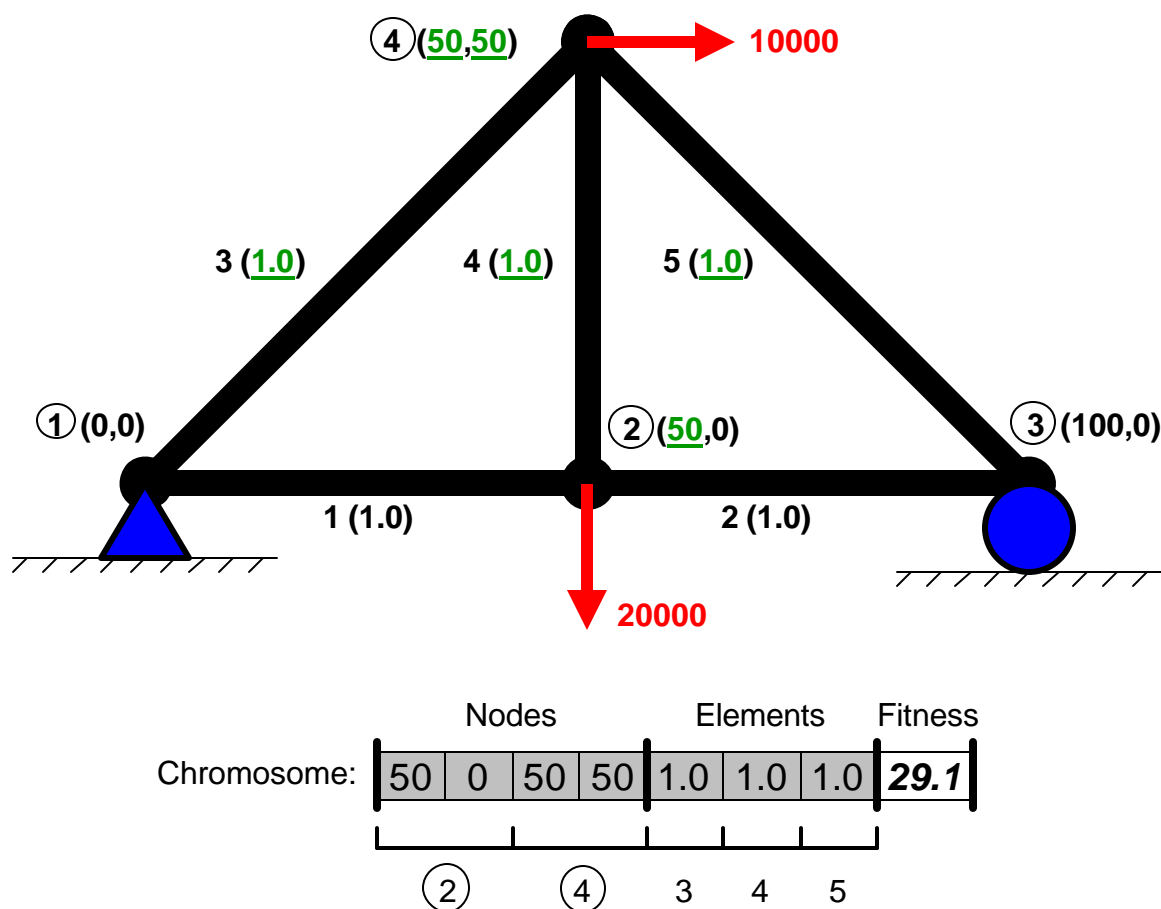


Figure 3.1.2. Structure-chromosome relationship

Recombination, shown in Figure 3.1.3, is the act of exchanging information between parents to create new and unique children. A *parent* is an individual containing

genetic information that is selected for recombination, while a *child* is the result of this recombination process. Recombination is often termed “mating”, “mixing”, “crossover”, or “xover”. This process consists of selecting two or more parents with the intent of creating one or more children, which is a slight variation of natural recombination where only one or two parents are allowed to create offspring. Figure 3.1.3 shows two parents being selected for recombination. The black circles represent loci to be left in place, while the white circles represent those to be swapped. The results of recombination applied to the selected loci are shown at the right of the figure. It is shown that three values have been swapped between the parent chromosomes to create the children. The one restriction to recombination in our application is that the chromosomes of the parents must be of the same form – they can’t replace node locations with frame section properties. This helps ensure that only valid offspring result from valid parents. Recombination promotes exploration of the search space in attempts to overcome local optima and discover better solutions. *Exploration* is a form of search that test unexplored regions of the search space (random search).

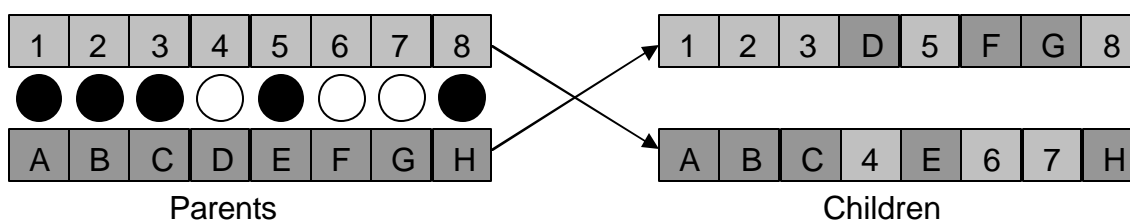


Figure 3.1.3. Visualization of recombination

Figure 3.1.4 shows an example of two structures performing recombination. Alleles within each chromosome are chosen to perform the crossover operation through random number generation. Note that the fitness values of the structures are shown in the

chromosome, but are not known to the genetic algorithm at this point. The arrows located on the nodes of the structures represent the two coordinate directions for each node.

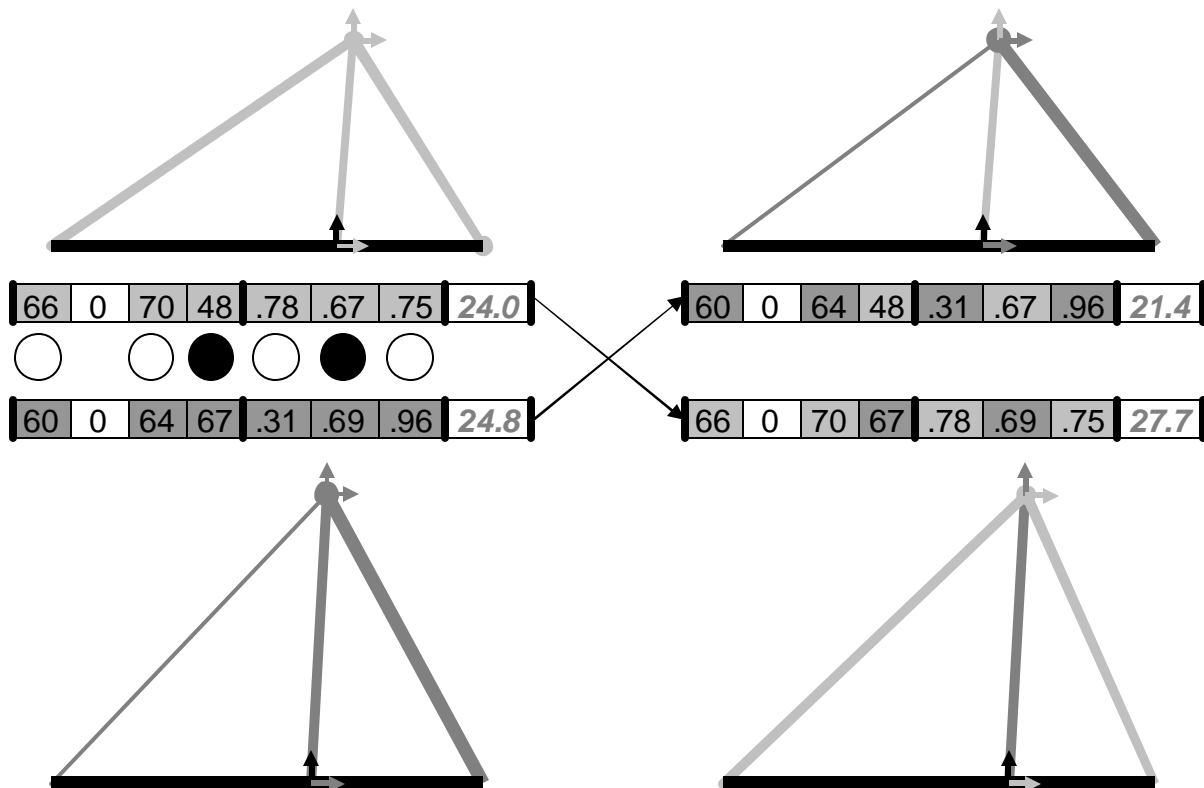


Figure 3.1.4. Example of recombination operator

Heredity is the idea that genetic information is passed from parents to children through every generation. This is important because it allows the survival of good genetic material in the path to optimal solutions. Children tend to retain at least some properties of one or more of their parents.

Variation, shown in Figure 3.1.5, is the mechanism used for introducing new genetic material into a population. It is often called “mutation” because a part of the chromosome is mutated or varied from its original state. Without variation, a genetic algorithm would only be able to find a solution containing some combination of the original

genetic material – severely limiting this method as an optimization process. Often, single alleles are perturbed or replaced with new values at some low probability as shown by the white locus in the right hand chromosome. This promotes exploitation of the solution in question by making only small changes in small amounts. *Exploitation* is a form of search that explicitly uses existing information to find better solutions (local search).

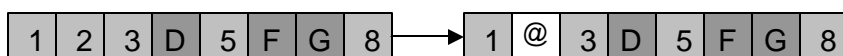


Figure 3.1.5. Visualization of variation

Figure 3.1.6 shows an example of the variation operator applied to the two children created in Figure 3.1.4. Again, the fitness value shown before the operation is not known to the algorithm. Notice that the variation of the chosen allele results in a worse fitness value. This happens sometimes, but may provide benefits later in the algorithm's execution. The fitness is not computed before the variation operator because of the large computational cost to the algorithm.

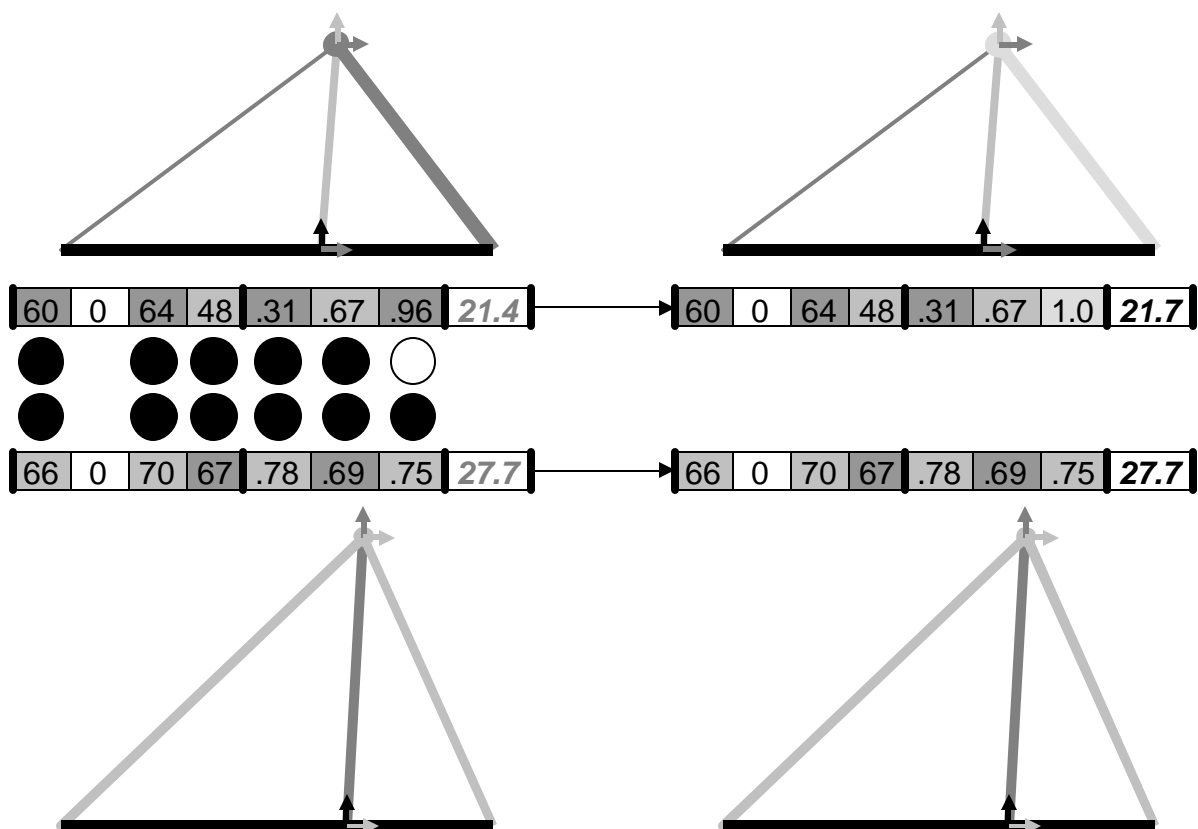


Figure 3.1.6. Example of variation operator

The mechanism for employing pressure and randomness to a population is called *selection*. This includes making a choice between multiple options and competing for survival. Selection is applied before recombination in order to choose parents, and defines the behavior of the population. Selective pressure is the driving force behind the optimization convergence. Too much selective pressure causes premature convergence and forces the algorithm to act as though it is a local search. Too little selective pressure causes near indefinite wandering and generally produces time-consuming and poor quality results.

3.2 Selected Methods and Parameters

While the basic framework of the GA (Genetic Algorithm) remains constant between specific applications, a multitude of parameters and directions are associated with an actual application of the method. Each step of the GA requires designer input and guidance in order to function effectively. The algorithm can be outlined to present a specific application as in Table 3.2.1. The table contains enough information to recreate the algorithm used, but not the information needed to recreate the representation or fitness function. The values in the right hand column are described in the content and equations of this section.

| | | |
|--------------------|--|---------------------------|
| Representation | Real & Integer Numbers | |
| Parent Selection | Linear Rank Based | Bias = 0.4 |
| Recombination | Uniform Allele Swap | Rate = 0.6 |
| Variation | Variable Perturbation | Rate = 0.05 |
| Survival Selection | Generational | |
| Specialties | Elitism Allele Symmetries Parsimony Pressure | Elites = 2 Weight = 10 |

Table 3.2.1. Outline of genetic algorithm used

The genetic algorithm in this work is different from other works in representation, selection, recombination, mutation, and specialty operators. Pyrz (2004) uses only discrete representations for truss optimization and heuristics-based mutation operators, severely limiting the algorithm's search capabilities. Hayalioglu (2001) uses a binary representation with a bit-flipping variation operator. The representation requires a more complex mapping function to obtain results, and the variation operator is completely random because it does not utilize information about the current design.

The structure is initially divided into constant and variable components – the individual chromosomes then consist only of values that are variable. This is done to minimize the amount of genetic material in order to reduce non-productive crossover and mutation operations. Before structural analysis can take place, the chromosome's genotype must be mapped to the phenotype space by combining variable values with constant values in the correct order and placed in the appropriate analysis files. Node location vectors and truss element areas are stored as real numbers, while beam element geometry is stored as a single integer that indexes a list of possible geometry sets. All three sets of data are stored in separate data structures so that they may be accessed independently. The data structures are allocated dynamically, which allows almost any number of variables to be used without user intervention. Every individual represents the variable information in the same order so that crossover occurs properly.

The basic execution of the genetic algorithm used is shown in the flow chart of Figure 3.2.1. Once the input values have been opened and read to memory as described in the text above, a large pool of random individuals is created analyzed and sorted. The individuals with the best fitness values are selected to create the first generation in the population. The top two individuals are then chosen to create copies of themselves to be inserted into the next generation. The remainder of the next generation is filled by choosing two parents at a time, performing recombination and variation on them, and inserting them into the next generation. Once the next generation is filled, each individual is analyzed and sorted according to a fitness value. If convergence has not occurred, the process is repeated with the new generation, otherwise a solution is said to be found. The content following the figure describes each step in more detail.

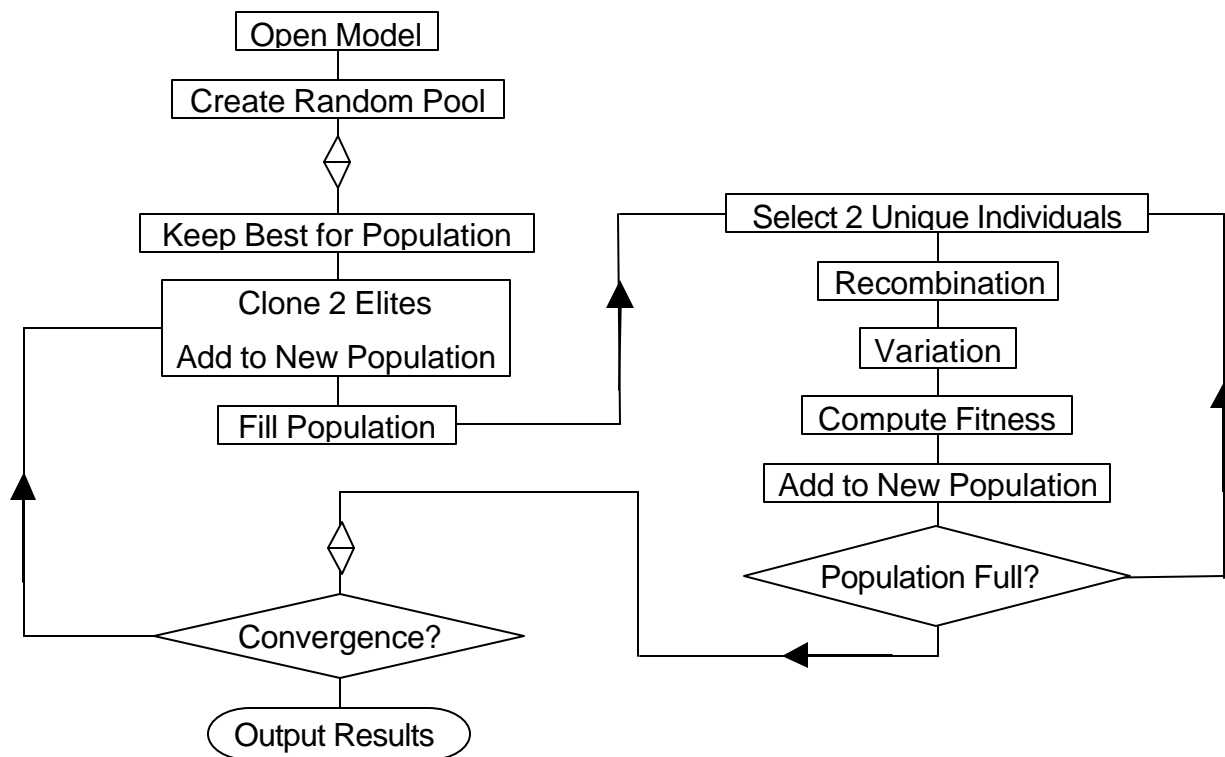


Figure 3.2.1. Flow chart of genetic algorithm

Parent selection is a linear biased rank based method that selects parents from a relative fitness ranking rather than absolute fitness value— who's better than whom, not how much. This method allows the possibility of selecting any individual (no matter how bad the fitness). The genetic algorithm also enforces unique parents to be selected for recombination. Parents are selected using (3.2.1), which is a variable linear bias equation derived from a linear function with a variable slope. The population size ($POPSIZE$) is multiplied by a distribution equation that uses a selection bias ($BIAS$) and a unit random number ($RANDOM$). Taking the floor of the result ensures that only integer values are computed ranging from 0 to ($POPSIZE-1$).

$$INDEX = \left\lfloor POPSIZE \cdot \frac{\sqrt{BIAS^2 + 4(1-BIAS) \cdot RANDOM} - BIAS}{2(1-BIAS)} \right\rfloor \quad (3.2.1)$$

Two unique parents are selected for recombination using the method described above so that recombination does not create two children identical to the two parents. The recombination mechanism is applied at a rate of 60% to every pair of parents in the population (60% of the parent sets will perform crossover). Those that do not perform crossover create clones of themselves, ensuring that their genetic material is passed on. Parents do have the opportunity to be chosen for recombination more than once per generation, and cannot recombine with themselves. This is often the case with the most-fit individuals in the current state. Recombination is performed by swapping alleles between parents at a probability of 50% applied to each allele. This produces a good mixing of variables and helps alleviate hitchhiking allele problems (“tag-alongs”).

Mutation is applied to every child or clone created after the recombination process with a low rate of 5%. This means that each allele in a chromosome has a 1 in 20 chance of mutation. Nodal positions and truss areas are perturbed some small amount through scaled addition using (3.2.2). This equation represents a polynomial curve crossing three points: (0,-1), (.5,0), and (1,1). This is made possible by splitting the polynomial equation into two parts and ensuring that no whole number exponents are used.

$$\begin{aligned}
 & \text{IF } \{ \text{RANDOM} < .5 \} \\
 & \text{THEN } \left\{ (2(.5 - \text{RANDOM}))^{MB+.00000001} \right\} \\
 & \text{ELSE } \left\{ (2(\text{RANDOM} - .5))^{MB+.00000001} \right\}
 \end{aligned} \tag{3.2.2}$$

As the mutation bias increases (MB), the amount of variation decreases in magnitude causing a longer but more refined search. Figure 3.2.2 shows the equation plot with $MB=3$ and the interaction between input and output values. The dots along the horizontal axis represent the unit random number [0,1) used in the equation. The dots

along the vertical axis represent the mutation value $[-1,1)$. It is apparent that the output values are not uniform in distribution and are collected toward zero. This defines a perturbation that can then be scaled to any variable of any magnitude.

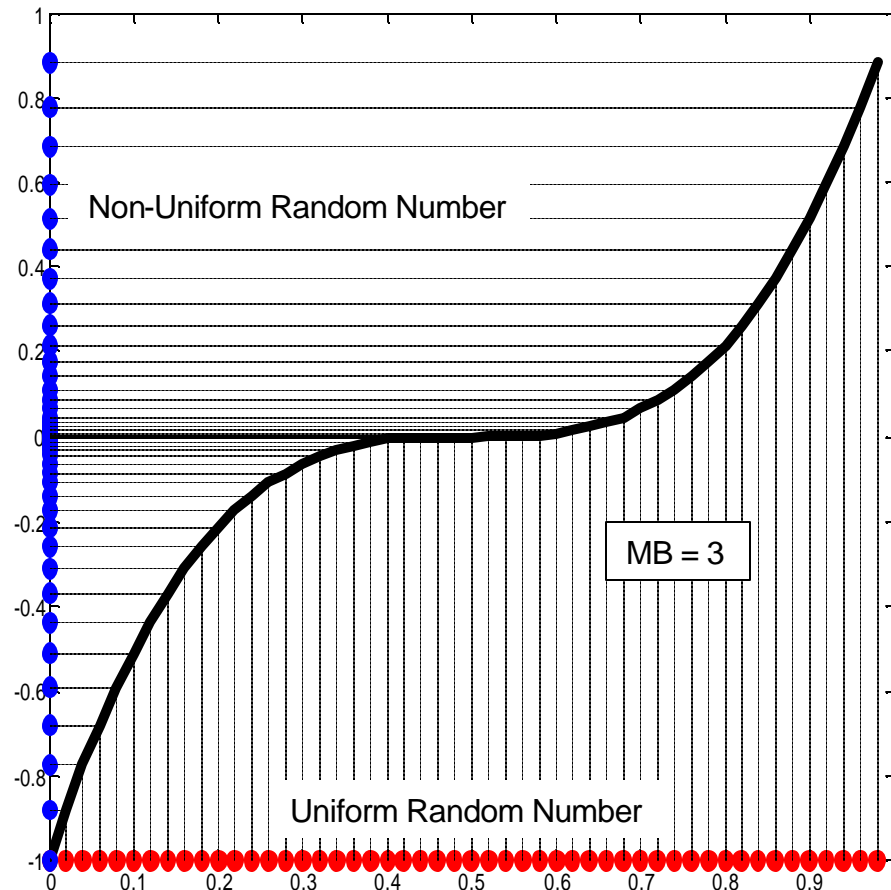


Figure 3.2.2. Graph of random number mapping

Mutation bias values are generally between 2 and 5 depending on the intent of the particular optimization problem. This scheme produces results close to a Gaussian distribution with varying levels of curvature. Figure 3.2.3 shows a probability distribution of random numbers as mutation bias increases. It is shown that as the mutation bias increases, output values are more concentrated toward zero. This means that it is more

likely for small changes to occur in the variation of an allele. A model utilizing high mutation bias focuses more on exploitation as its search method.

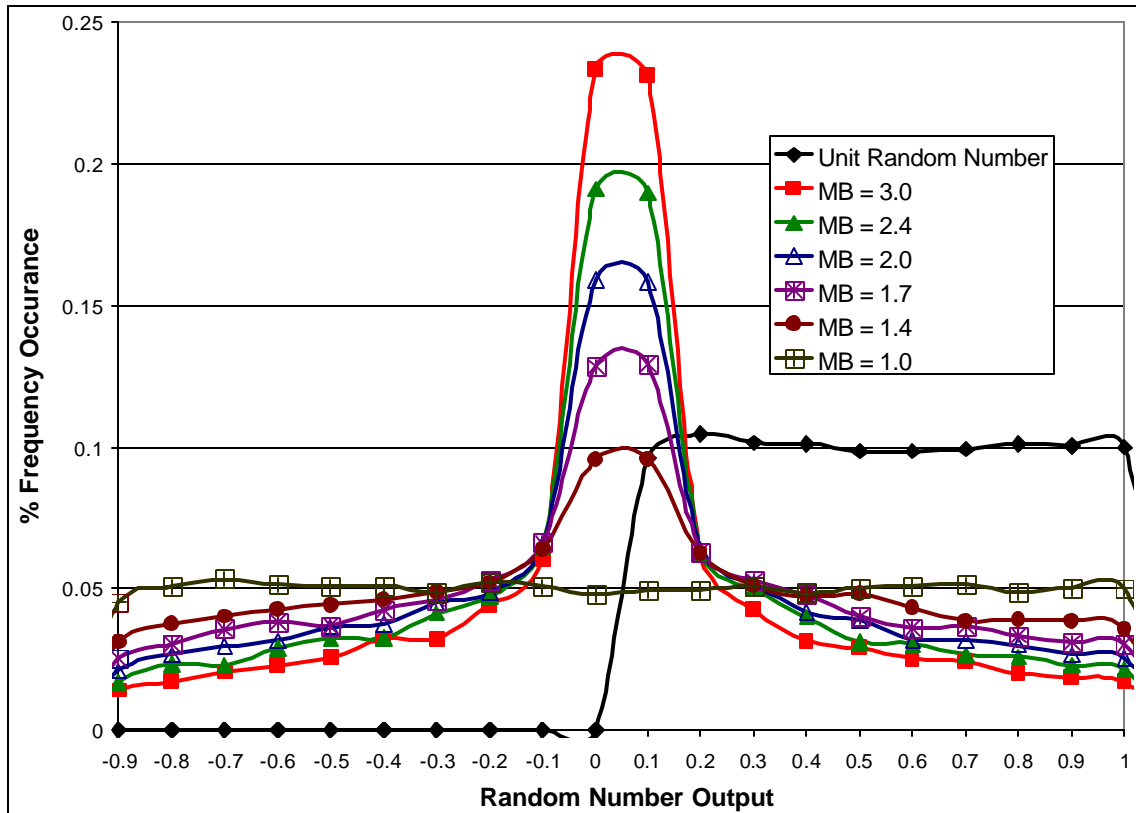


Figure 3.2.3. Random number variation graph

The generational survival selection method is a simplified version of nature's model. One population creates a new generation by selecting two parents at a time and performing recombination and variation as described. The new generation is then used to create the next generation and all memory of the previous generation is lost. This results in highly diverse populations that take longer to converge to a solution. Premature convergence can result in sub-optimal results in highly complex search spaces due to the fact that a wide variety of solutions have not been explored. The problem with the

generational algorithm is that good genetic information can be lost due to the replacement of populations.

The loss of information is modified by introducing the idea of *elitism*. Before offspring are created for the next generation, the two best individuals of the current population are selected to create copies of themselves into the next generation. This insures that their genetic information is not lost, while also leaving them the opportunity to be selected as parents. The idea of *convergence*, as mentioned previously, occurs when the top individual remains most fit for a certain number of generations. A typical value used lies between 100 and 500 generations of consecutive elitism. As the convergence limit increases, so does the search time and quality of search.

Another specialty added to the standard GA is allele symmetries. This method allows the reduction of search space due to the minimization of information. It allows one variable to be subjected to changes, and causes other variables to follow the value of this variable. Mathematically, it is similar to the system of equations (3.2.3).

$$\begin{aligned} &mutate(X1) \\ X2 &= -X1 \\ X3 &= X1 \end{aligned} \tag{3.2.3}$$

This means that structures may be forced to nodal symmetries across coordinate planes. Chosen elements may also exhibit identical cross sectional geometry to other elements included in the optimization process.

This shows three design variables essentially represented by the optimization of one. The idea of information minimization is important when hundreds of variables can be optimized by reduction to several dozen. This reduces search spaces from hundreds of

dimensions to many magnitudes smaller, while also reducing the number of local optima for the GA to get stuck in.

Parsimony pressure is the last specialty added to the GA. This is the mechanism that modifies the fitness of a structure based on constraints placed on output values – it penalizes “bad” structures. The fitness analysis is discussed in more detail later in the paper.

3.3 Optimization Capabilities

The structural genetic algorithm has the capability of varying the basic size and shape parameters for a given structure. These parameters include nodal locations in any combination of the three dimensions, symmetries of nodes about coordinate planes, element sizes, and symmetry of element cross sectional properties. For parsimony pressure, limits can be placed on worst element stress and nodal displacement in any direction or rotation. The software developed also has the capability to display the shape of the structure in an interactive 3D space, giving the user a very good visual representation of the models. Another visual tool displays the fitness history of the four best individuals as the problem is driven to convergence.

3.4 Fitness Analysis

The fitness of a structure is based primarily on weight with the goal of minimizing this fitness. The parsimony pressure adds some value to the fitness, resulting in a “heavier” structure. Stress parsimony pressure is based on user-defined yield strength of the material(s), and is not optional. Displacement parsimony pressure is based on a set of user-defined boundaries for each node in each direction and rotation, and is optional. For

example, a displacement limit may be set for all nodes in all directions or a subset of nodes in a chosen direction. The fitness, adapted from Hayalioglu (2001), is calculated using the system of equations shown in (3.4.1).

$$FITNESS = MASS \cdot (1 + dW \cdot dV + sW \cdot sV) \quad (3.4.1)$$

The weighting constants for displacement (dW) and stress (sW) are multiplied by displacement violations (dV) and stress violations (sV) to achieve a weighted parsimony pressure. The displacement and stress violation values are calculated using (3.4.2) and (3.4.3).

$$dV = \sum_{l=1}^{NL} \sum_{n=1}^{NN} D_{l,n} \quad (3.4.2)$$

$$sV = \sum_{l=1}^{NL} \sum_{e=1}^{NE} S_{l,e} \quad (3.4.3)$$

Note that the displacement violation magnitudes ($D_{l,n}$) and stress violation magnitudes ($S_{l,e}$) are summed over all loading cases (NL), nodes (NN), and elements (NE) where applicable. To ensure only limit violations contribute to the parsimony pressure, (3.4.4) and (3.4.5) are used.

$$D_{l,n} = \max \left\{ 0, \left(\frac{displacement_{l,n}}{dLIMIT_n} - 1 \right) \right\} \quad (3.4.4)$$

$$S_{l,e} = \max \left\{ 0, \left(\frac{stress_{l,e}}{sLIMIT_e} - 1 \right) \right\} \quad (3.4.5)$$

The displacement limit ($dLIMIT_n$) for each node (n) is defined by the user in the optimization input file as an upper boundary for chosen degrees of freedom, and the stress

limit ($sLIMIT_e$) for each element (e) is defined by the yield strength of the material associated with the element. The values for nodal displacements ($displacement_{i,n}$) and element stresses ($stress_{l,e}$) are drawn from the structural matrix analysis.

The set of fitness functions require three outputs from the structural analysis: weight, stress, and displacement. All other outputs from the analysis software are suppressed for use with the optimization routine. A weighting value must be supplied by the user that determines how important the boundary conditions are. A value of 10 is often used as a baseline and produces structures that do not violate constraints by any amount. If the weighting values are too low, the structures produced might frequently overstep boundaries. But if the weighting values are too high, structures may converge to sub-optimal solutions because of an over-increased selective pressure. The adjusted fitness value is used to rank each individual in the population, which can then be used in the rank based parent selection.

These sets of equations result in a positive value representative of the magnitude of violation. They will never reward the structure for not violating constraints because displacement and stress are inversely proportional to mass, so rewarding these values will cause heavy and rigid structures to result because of higher attained fitness values. In general, the fitness of a structure is equal to or greater than it's mass.

3.4.1 Optimization Software Used

Five programs were created using the C++ and C languages to provide portability and speed of execution. The software includes programs for input file template generation, graphical optimization history viewing, 3D graphical viewing and interaction for structures,

structural analysis, and structural optimization. All software uses the same set of input files in order to reduce user effort. There are five input files required to use this optimization software package: nodes, elements, materials, loads, and optimization parameters. All input and output files are CSV files (comma separated value) that can be edited easily in either a spreadsheet application or simple text editor. Figure 3.4.1 shows the structural optimization program (left), the graphical viewing utility (upper right), and the graphical optimization history utility (lower right). It should be noted that the fitness history shown in the figure is typical for genetic algorithms, with substantial progress made at the start of the search and very little at the end.

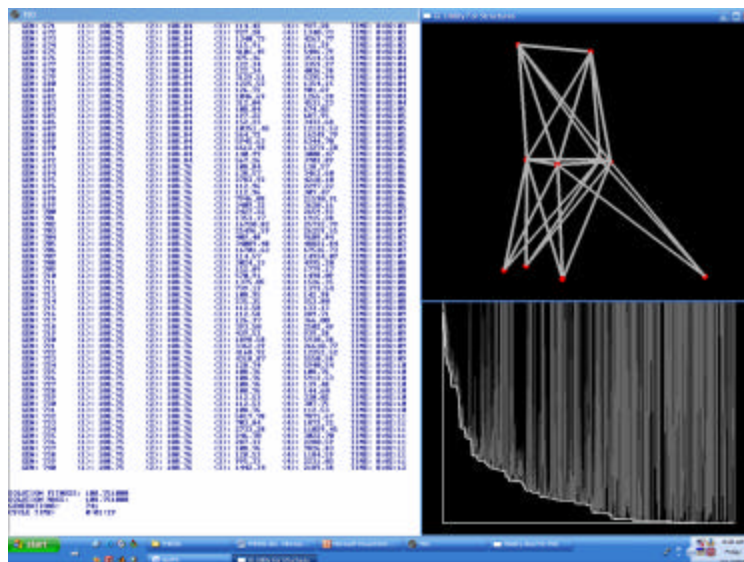


Figure 3.4.1. Screenshot of optimization software

3.4.2 Analysis Capabilities

The structural analysis software has all of the basic tools needed for linear elastic analysis. The most important feature of this software is that its results have been compared to ALGOR® and MASTAN2 commercial software with 100% accuracy to six significant digits. It can handle 2D or 3D structures made up of truss and/or frame

elements so long as the necessary boundary conditions are placed on the nodes of the structure. The beam and truss elements are contained in the same input file and designated by a number in the description properties. Beam element geometry may be defined either by direct property values or by designating a pre-defined section and supplying the necessary dimensions. The beam strong and weak axis orientations are either set by the software if not specified or can be designated in any one of the three coordinate directions. Loads are applied only at nodes in any combination of six directions. Multiple loading cases are also supported, creating multiple output sets.

Figure 3.4.2 shows the execution flow used in the structural matrix analysis software. The method is based on the material presented in McGuire (2000). It begins by reading all input values including: node locations, nodal constraints, element connectivity, element properties, applied loads, and material properties. Element length, volume, and mass is then calculated and written to a file. A local coordinate stiffness matrix for each element is calculated using material and physical properties. Then, transformation matrices are calculated based on each element's orientation with respect to the global coordinate system. The element stiffness matrices are then transformed to the global coordinate system and assembled into a single global stiffness matrix. This matrix is then reduced by removing the constrained degrees of freedom. In a traditional structural matrix analysis, the reduced global stiffness matrix is inverted to produce a global flexibility matrix. This work uses matrix decomposition to solve the force-displacement equations, which is described in the content of this section discussing speed of execution. Once the matrix has been decomposed, loads may be applied and displacements calculated relative to the global coordinate system. In order to compute element internal forces, the displacements

must be transformed back to local coordinates for each element. The internal forces are then used to calculate element stresses, which are the last results to be output. The calculation of displacement, force, and stress is completed for each loading condition.

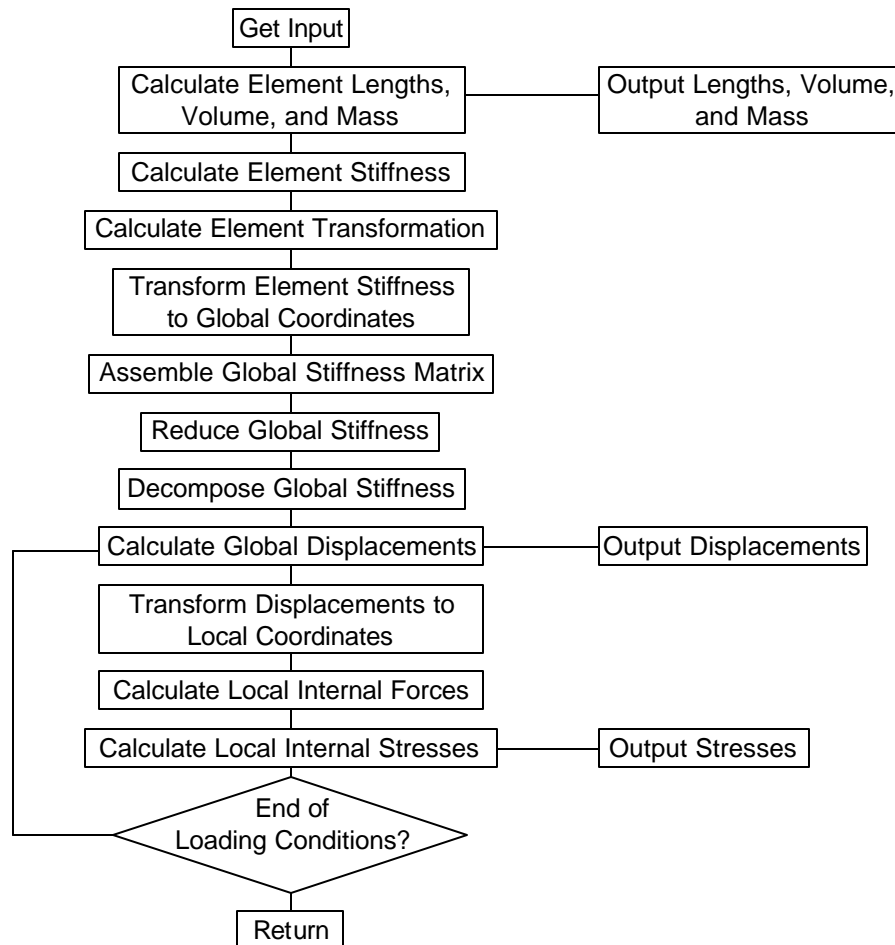


Figure 3.4.2. Flow chart of structural analysis

The structural analysis can be run with a combination of options. *Verbose* mode outputs each step taken and if any errors occurred. The graphics utility may also be called before and/or after analysis using command line options. The remaining command options designate what is to be solved for and output to CSV files. By default, the software outputs element and global properties (length, volume, mass). There are options to output the structure's global, reduced, and reduced inverse stiffness matrices for problem tracking

and debugging. The user may also wish to view output properties such as nodal displacements, nodal reaction forces, internal element forces, and element stresses (including worst combination of axial and bending).

Speed of execution is the second most important feature of the structural analysis software when used in conjunction with an evolutionary optimization routine because it may need to execute several dozen times for a single generation to occur. The speed of the software has been recorded at over 80 executions per second with an 18 DOF model and 1.5 executions per second with a 744 DOF model -- within the optimization software.

The initial structural analysis used matrix inversion to solve for displacements. After profiling the software for execution time, it was found that the matrix inversion required the most execution time. Several alternative methods for matrix inversion were reviewed from McGuire (2000), including Gaussian elimination, the Cholesky method, and the Doolittle method. Upon profiling with MatLAB®, it was found that a compact form of Gaussian elimination was computationally less expensive. This method consists of matrix decomposition, forward, and backward substitution as presented by McGuire (2000).

The equations for matrix decomposition are shown in (3.4.6). They decompose a symmetric positive definite square matrix into an upper triangular matrix, a diagonal matrix, and a lower triangular matrix. The original method presented by McGuire (2000) uses separate matrices for the upper triangular, lower triangular, and diagonal matrices. The compact method derived allows the original matrix to be transformed without allocating new memory for new matrices, which is also a time saving feature. The first equation computes the lower triangular matrix, while the second equation computes the diagonal

and upper triangular matrices. The variables i , j , and a are used as counters in the ranges shown in the equations. The variable n denotes the size of the matrix, and matrix indexing begins with zero.

$$\begin{aligned}
 & & & i = 0 \rightarrow n-1 \\
 K_{ji} &= \frac{K_{ij}}{K_{jj}} & & j = 0 \rightarrow n-1 \\
 K_{ji} &= K_{ij} - \sum_{a=0}^{i-1} (K_{ai} \cdot K_{ja}) & & j = i \rightarrow n-1
 \end{aligned} \tag{3.4.6}$$

After the matrix has been decomposed into its components, a series of substitutions are applied using the load vector (F). Equation (3.4.7) shows the process of forward and backward substitution, also found in McGuire (2000). The first step uses the lower triangular matrix, the diagonal matrix, and the load vector to determine an intermediate vector (Y). The backward substitution then uses the upper triangular matrix and the intermediate vector (Y) to calculate the displacement vector (U).

$$\begin{aligned}
 Y_i &= \frac{F_i - \sum_{j=0}^{i-1} (K_{ij} \cdot Y_j)}{A_{ii}} & & i = 0 \rightarrow n-1 \\
 U_i &= Y_i - \sum_{j=i+1}^{n-1} (K_{ij} \cdot U_j)
 \end{aligned} \tag{3.4.7}$$

4.0 BENCHMARK STUDIES

Three studies were performed to evaluate the software package’s capabilities. The first is a benchmark 2D 10-bar truss with a single loading case that is commonly used to verify new structural optimization techniques. The second case is a classic 3D 25-bar truss problem with two loading cases that has been a benchmark since 1969. The last study conducted was a miniature formula-style race car frame used in the Formula SAE collegiate competition. The first two cases show the validity of the technique presented through comparison with studies conducted in the literature and with GENESIS® commercial structural optimization software. The last case shows the feasibility to application on real world problems.

The problems were solved using a set procedure shown in Figure 4.0.1. This procedure was employed to minimize the amount of designer intervention and to limit the amount of time spent on each problem. It also gave the software the opportunity to better explore the design space by restarting the search many times. With regard to the figure, an “complete optimization” occurs when the software ends execution and a solution is presented, while a “trial” is an internal restart without presenting a solution.

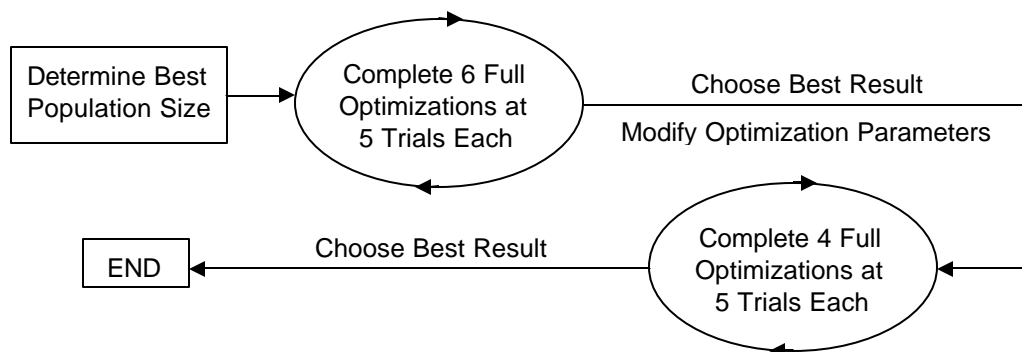


Figure 4.0.1. Optimization Procedure Used

The 25-bar truss benchmark was also used to study the effects of design variable reduction with this genetic algorithm. This is achieved through the use of nodal and elemental symmetries in various combinations. A total of six unique problems were optimized, each with a different number and type of design variables. Conclusions are drawn on this topic in section 4.2.3 after the comparison of results.

4.1 10-Bar Truss Benchmark

The object of the 10-bar truss benchmark is to compare results to a simple, well-defined structure with few variables and constraints. While Pyrz (2004) presents this problem in the literature, it was not chosen for results comparisons because discrete variables were used for design variables. The comparative work (Romero, 2004) presents all necessary information to reconstruct the problem and the structure was analyzed using linear elastic techniques. This structure is optimized for minimization of mass with a single loading case and stress constraints applied to every member. Displacement constraints are not considered for this problem. A single material is used that exhibits the properties of aluminum.

The dimensions, node numbers, element numbers, and loaded nodes for this structure are shown in Figure 4.1.1. The single load case values are shown in Table 4.1.1 and the material properties for the entire structure are shown in Table 4.1.2. Ten design variables are considered for optimization (each element cross sectional area) with a minimum value limit of 0.1 in^2 and an initial value of 3.0 in^2 . The initial weight of the structure is 4196.47 lb.

Two studies were performed on this structure: a traditional size-only optimization and a size + shape optimization. The size-only optimization is compared with the results found by Romero (2004) and those found by GENESIS®. The size + shape optimization is only compared with the results found by GENESIS® because the literature did not present this problem. It was conducted to show the effects of combining optimization techniques.

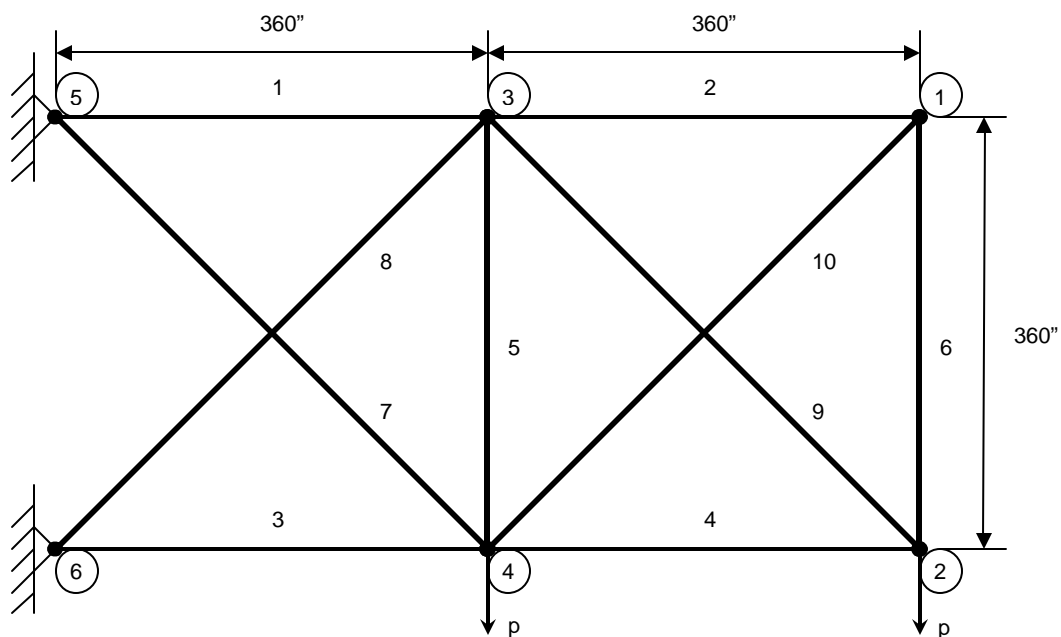


Figure 4.1.1. 10-Bar Truss

| Node | Load Case 1 (lb) | | |
|------|------------------|--------|---|
| | x | y | z |
| 2 | 0 | 100000 | 0 |
| 4 | 0 | 100000 | 0 |

Table 4.1.1. 10-Bar Truss Loads

| Material | E | n | s_y | g |
|----------|--------------|------|-----------|------------------------|
| Aluminum | 10000000 psi | .334 | 25000 psi | 0.1 lb/in ³ |

Table 4.1.2. 10-Bar Truss Material

4.1.1 Comparison of Results

Three solutions are shown in table 4.1.3: GA software, GENESIS®, and Romero. Each solution presents the optimal element area and stress. Stress values displayed in bold are in violation of stress constraints. The solution mass is also shown with the fitness value found by the GA. Solutions that violate stress constraints have a fitness value higher than the mass. The solution presented by Romero (2004) has a mass lower than that found by the GA, but at the cost of stress violations. When comparing fitness values, the genetic algorithm outperformed the other solutions.

| Element | Auer (2005) | | GENESIS® (2005) | | Romero (2004) | |
|-----------|-------------------------|--------------|-------------------------|--------------|-------------------------|-----------------|
| | Area (in ²) | Stress (psi) | Area (in ²) | Stress (psi) | Area (in ²) | Stress (psi) |
| 1 | 7.9324 | 24999.6 | 7.961 | 24927.5 | 7.9378 | 25000.2 |
| 2 | 0.1 | 16932.7 | 0.1 | 15494.9 | 0.1 | 15533.4 |
| 3 | 8.0678 | -24999.7 | 8.0872 | -24922.4 | 8.0621 | -25000.1 |
| 4 | 3.9323 | -25000.0 | 3.9490 | -24930.3 | 3.9378 | -25000.4 |
| 5 | 0.1 | -0.5 | 0.1 | -25.0 | 0.1 | -0.6 |
| 6 | 0.1 | 16932.7 | 0.1 | 15494.9 | 0.1 | 15533.4 |
| 7 | 5.7527 | 24999.7 | 5.7632 | 24919.7 | 5.7447 | 25000.1 |
| 8 | 5.5611 | -25000.0 | 5.5848 | -24929.7 | 5.5689 | -25000.4 |
| 9 | 5.5611 | 25000.0 | 5.5848 | 24930.3 | 5.5689 | 25000.4 |
| 10 | 0.1164 | -20567.5 | 0.1 | -21913.1 | 0.1 | -21967.5 |
| Fitness | 1593.420000 | | 1597.860000 | | 1594.179622 | |
| Mass (lb) | 1593.42 | | 1597.86 | | 1593.16 | |

Table 4.1.3. 10-bar truss size-only optimization results

Two solutions are presented for the size + shape optimization of the 10-bar truss: GA software and GENESIS®. In this study, nodes one and three were bounded to the xy-plane, node five was bounded to the y-axis, and the remaining nodes were fixed in location. Table 4.1.4 shows the solution found by the GA, including element area and stress with nodal positions and the solution fitness and mass. Figure 4.1.2 shows the shape of the solution found by the GA. Note that the addition of shape optimization further reduces the mass of the structure by over 350 lb.

| Element | Area (in ²) | Stress (psi) | Node | X-Pos (in) | Y-Pos (in) | | | | |
|------------|-------------------------|--------------|---|------------|------------|----------|---------|------------|---------|
| 1 | 4.8847 | 24999.2 | 1 | 642.427 | 143.557 | | | | |
| 2 | 0.1 | 24693.5 | 2 | 720 | 0 | | | | |
| 3 | 4.1691 | -24999.7 | 3 | 523.361 | 371.946 | | | | |
| 4 | 2.1157 | -24999.8 | 4 | 360 | 0 | | | | |
| 5 | 0.1 | 5466.0 | 5 | 0 | 694.251 | | | | |
| 6 | 0.1 | 24700.0 | 6 | 0 | 0 | | | | |
| 7 | 4.4825 | 24999.8 | <table border="1"> <tr> <td>Fitness:</td> <td>1236.46</td> </tr> <tr> <td>Mass (lb):</td> <td>1236.46</td> </tr> </table> | | | Fitness: | 1236.46 | Mass (lb): | 1236.46 |
| Fitness: | 1236.46 | | | | | | | | |
| Mass (lb): | 1236.46 | | | | | | | | |
| 8 | 2.5193 | -24999.4 | | | | | | | |
| 9 | 4.4264 | 24999.1 | | | | | | | |
| 10 | 0.1 | 366.8 | | | | | | | |

Table 4.1.4. 10-bar truss size + shape optimization results for GA

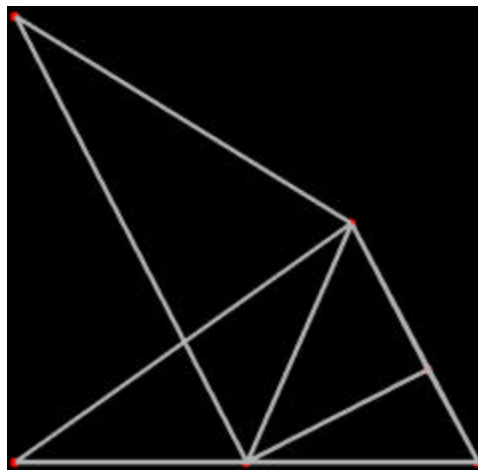


Figure 4.1.2. 10-bar truss shape found by genetic algorithm

Table 4.1.5 shows the solution found by GENESIS®, including element area and stress with nodal positions and the solution fitness and mass. Figure 4.1.3 shows the shape of the solution found by the software. Appendix A holds the input files for the initial structure, an intermediate solution, and the full final solution found by the GA. Note that GENESIS® found a solution nearly four pounds lighter than the solution found by the GA. This is due to the fact that the GENESIS® solution is far less robust than the GA solution, meaning that a small deviation from any of the element areas or nodal locations cause relatively large changes in the fitness.

| Element | Area (in ²) | Stress (psi) | Node | X-Pos (in) | Y-Pos (in) | | | | |
|------------|-------------------------|--------------|---|------------|------------|----------|--------|------------|--------|
| 1 | 4.8322 | 24923.2 | 1 | 686.284 | 130.621 | | | | |
| 2 | 0.27127 | 24999.2 | 2 | 720 | 0 | | | | |
| 3 | 4.0502 | -24916.6 | 3 | 540.459 | 355.233 | | | | |
| 4 | 1.9632 | -24926.9 | 4 | 360 | 0 | | | | |
| 5 | 0.10003 | 4990.8 | 5 | 0 | 719.865 | | | | |
| 6 | 0.26833 | 24999.0 | 6 | 0 | 0 | | | | |
| 7 | 4.5035 | 24916.5 | <table border="1"> <tr> <td>Fitness:</td> <td>1232.8</td> </tr> <tr> <td>Mass (lb):</td> <td>1232.8</td> </tr> </table> | | | Fitness: | 1232.8 | Mass (lb): | 1232.8 |
| Fitness: | 1232.8 | | | | | | | | |
| Mass (lb): | 1232.8 | | | | | | | | |
| 8 | 2.3597 | -24906.1 | | | | | | | |
| 9 | 4.2159 | 24851.1 | | | | | | | |
| 10 | 0.10001 | -21716.0 | | | | | | | |

Table 4.1.5. 10-bar truss size + shape optimization results for GENESIS®

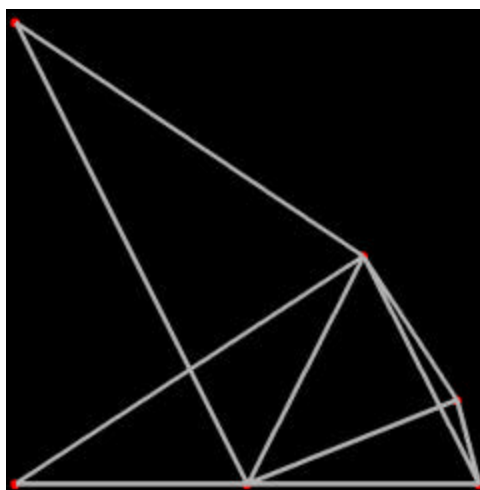


Figure 4.1.3. 10-bar truss shape found by GENESIS®

The precision of results found by the GA shows that the method is capable of producing results comparable to the literature and commercial software. The accuracy of the results were proven by using the input values presented in the literature and GENESIS®, and comparing the presented results to those calculated by the genetic algorithm. The mass values were found accurate to 0.03 lb in a structure weighing 1232 lb between analysis packages. The validity of the shape optimization is also reassured by the similar shape results found with GENESIS®.

4.2 25-Bar Truss Benchmark

While Azid (2002), Ülker (2001), and Sunar (2001) present this problem in the literature, their work was not used for comparison of results because they either did not present enough information to reconstruct the problem or they did not also present the 10-bar truss problem. Romero (2004) was used again in the comparison study for reasons stated previously. Multiple comparisons of this problem were not possible because each piece of literature uses a different problem description.

The object of the 25-bar truss benchmark is to compare results to a simple, well defined structure with many variables, constraints, and loading conditions. This structure is optimized for minimization of mass with two loading cases and stress constraints applied to every member. Displacement constraints of 2.0 inches are applied at every node in all three coordinate directions. A single material is used that exhibits the properties of aluminum.

The dimensions, node numbers, element numbers, and constraints for this structure are shown in Figure 4.2.1. The two load case values are shown in Table 4.2.1 and the material properties for the entire structure are shown in Table 4.2.2. The design variables consist of element cross sectional areas and nodal positions, but the details of these variables differ between the six studies conducted and are discussed later. When element areas are considered for optimization, a minimum value limit of 0.01 in^2 and an initial value of 3.0 in^2 is used. The initial weight of the structure is 992.162 lb. The initial fitness of the structure is also 992.162 lb due to zero stress and displacement violations.

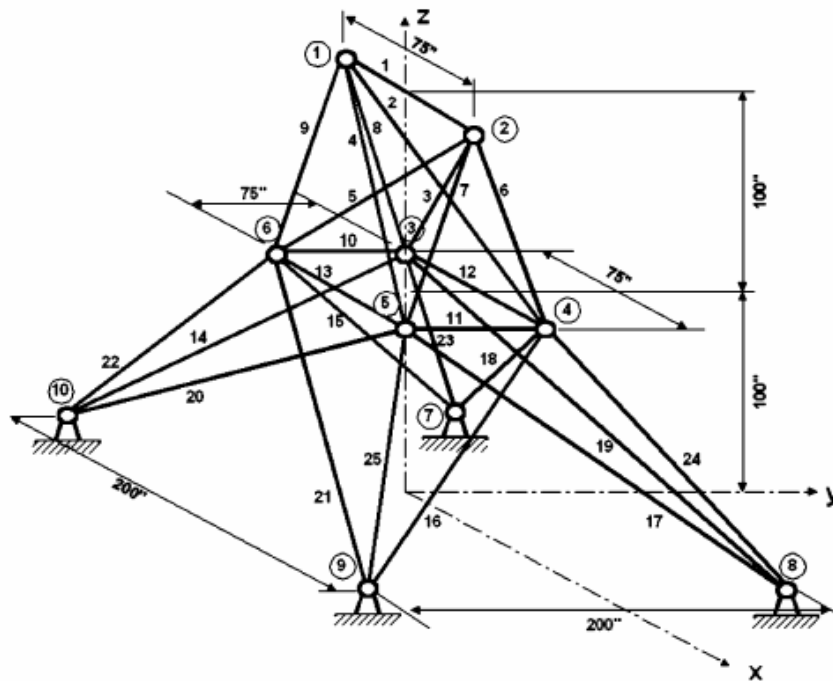


Figure 4.2.1. 25-bar truss

| Node | Load Case 1 (lb) | | | Load Case 2 (lb) | | |
|------|------------------|-------|-------|------------------|-------|-------|
| | x | y | z | x | y | z |
| 1 | 1000 | 10000 | -5000 | 0 | 20000 | -5000 |
| 2 | 0 | 10000 | -5000 | 0 | 20000 | -5000 |
| 3 | 500 | 0 | 0 | 0 | 0 | -5000 |
| 6 | 500 | 0 | 0 | 0 | 0 | -5000 |

Table 4.2.1. 25-Bar Truss Loads

| Material | E (psi) | n | s_y (psi) | g (lb/in ³) |
|----------|-----------|------|-------------|---------------------------|
| Aluminum | 10000000 | .334 | 35294 | 0.1 |

Table 4.2.2. 25-Bar Truss Material

Six studies were performed on this structure and are outlined in Table 4.2.3. The table enumerates the studies while conveying the type of optimization presented and the corresponding number of design variables. Study 1 is compared with the results found in the literature and those found by GENESIS®. The remaining five studies are only compared with the results found by GENESIS® because the literature did not present

these problems. Six studies were conducted to show the effects of combining optimization techniques and the effects of design variable reduction.

| Study | Size Optimization | Size Symmetry | Shape Optimization | Shape Symmetry | Design Variables |
|-------|-------------------|---------------|--------------------|----------------|------------------|
| 1 | X | X | | | 8 |
| 2 | X | | | | 25 |
| 3 | X | X | X | X | 12 |
| 4 | X | X | X | | 24 |
| 5 | X | | X | X | 29 |
| 6 | X | | X | | 41 |

Table 4.2.3. 25-bar truss studies

Size optimization is applied to all six studies, half of which utilize size symmetry. Studies not utilizing size symmetry are considered to have 25 size variables, one for each element cross sectional area. All 25 of these variables are independent of each other and may hold unique values. The studies using size symmetry group element cross sectional areas according to Table 4.2.4. This use of symmetry implies that all elements grouped under a design variable must maintain the same cross sectional area, but each grouping of elements is allowed to vary independently.

| Variable | Element |
|----------|---------|
| 1 | 1 |
| 2 | 2-5 |
| 3 | 6-9 |
| 4 | 10-11 |
| 5 | 12-13 |
| 6 | 14-17 |
| 7 | 18-21 |
| 8 | 22-25 |

Table 4.2.4. Size symmetry variables

Shape optimization is applied to the last four studies, half of which utilize shape symmetry. Studies not using shape symmetry are considered to have 16 shape variables. Nodes 1 and 2 remain unvarying in all studies. Nodes 3 through 6 are permitted to move in

any direction parallel to the xy -plane at an elevation of 100 inches, using two shape variables for each node. Nodes 7 through 10 are permitted to move in any direction along the xy -plane, also using two shape variables for each node. Shape variable boundaries are placed at the coordinate xz and yz planes, which prevents nodes from occupying the same location. The studies using shape symmetry follow the rules of Table 4.2.5, effectively reducing 16 variables to four. The table shows the procedure used to set the shape of the structure. First, nodes 4 and 8 are varied in the x and y directions according to the genetic algorithm. Then, each line in the table is executed, which sets two more dependent shape variables per line. After the symmetries are applied, four independent shape variables and 12 dependent shape variables are set. The same allowable movements and boundaries as the non-symmetric studies are enforced. This use of symmetry produces a structure that is symmetric about the xz and yz coordinate planes.

| Node | Symmetric With | About Plane |
|------|-------------------|----------------|
| 3 | 4 | yz |
| 5 | 4 | xz |
| 6 | 3 | xz |
| 7 | 8 | yz |
| 9 | 8 | xz |
| 10 | 7 | xz |

Table 4.2.5. Shape symmetry rules

4.2.1 Comparison of Results

Three solutions to study 1 are shown in Table 4.2.6: GA software, GENESIS®, and Romero. Each solution presents the structural mass and the fitness value found by the genetic algorithm software. The element areas, element stresses, and nodal displacements are not presented because of the large volume of numbers that would be needed. Solutions that violate stress or displacement constraints have a fitness value

higher than the mass. The solution presented by Romero has a mass lower than that found by the genetic algorithm software, but at the cost of large stress and displacement violations that lead to a much higher fitness value. It appears as if the constraints set by Romero were disregarded to great extent, and his solution provides no reasonable comparison to the other results. When comparing fitness values and mass values, the genetic algorithm performed within 0.1 lb of GENESIS®.

| | Auer | GENESIS® | Romero |
|-----------|---------|----------|----------|
| Fitness | 160.009 | 159.902 | 6772.780 |
| Mass (lb) | 160.009 | 159.902 | 100.089 |

Table 4.2.6. 25-bar truss results – study 1

Two solutions to studies 2 through 6 each are presented in Table 4.2.7: those found with the genetic algorithm and GENESIS®. Note that GENESIS® outperformed the genetic algorithm by several pounds in all but one study. Appendix B holds data for the initial structure, intermediate results, and final results to all six studies.

| Study | Auer | | GENESIS® | |
|-------|---------|-----------|----------|-----------|
| | Fitness | Mass (lb) | Fitness | Mass (lb) |
| 2 | 163.295 | 163.295 | 162.173 | 162.173 |
| 3 | 60.891 | 60.891 | 62.974 | 62.871 |
| 4 | 67.646 | 67.646 | 62.004 | 61.986 |
| 5 | 58.530 | 58.530 | 51.838 | 51.838 |
| 6 | 56.171 | 56.171 | 51.345 | 51.345 |

Table 4.2.7. 25-bar truss results – studies 2 through 6

Many of the genetic algorithm's solutions to studies utilizing shape optimization have a drastically different shape than the solutions found by GENESIS®. Figures 4.2.2 through 4.2.5 show the comparison of shape between the two software solutions for studies 3, 4, 5, and 6, respectively. Note that studies 3 and 5 use shape symmetry.

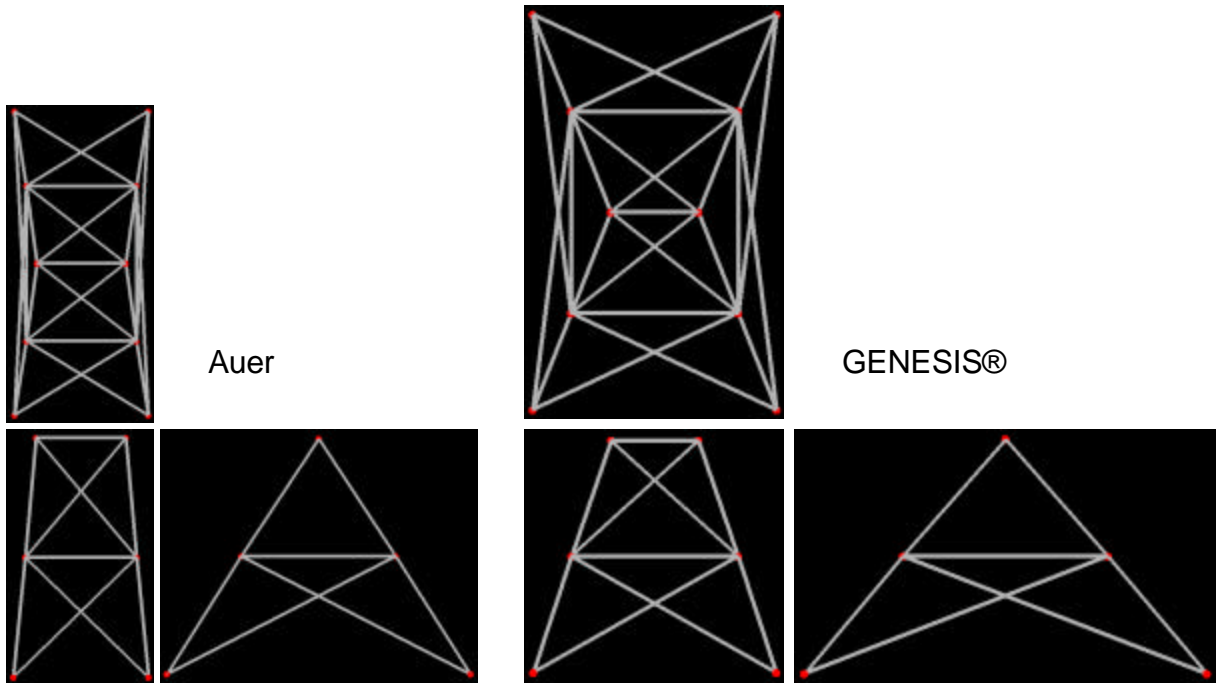


Figure 4.2.2. Shape solutions to study 3

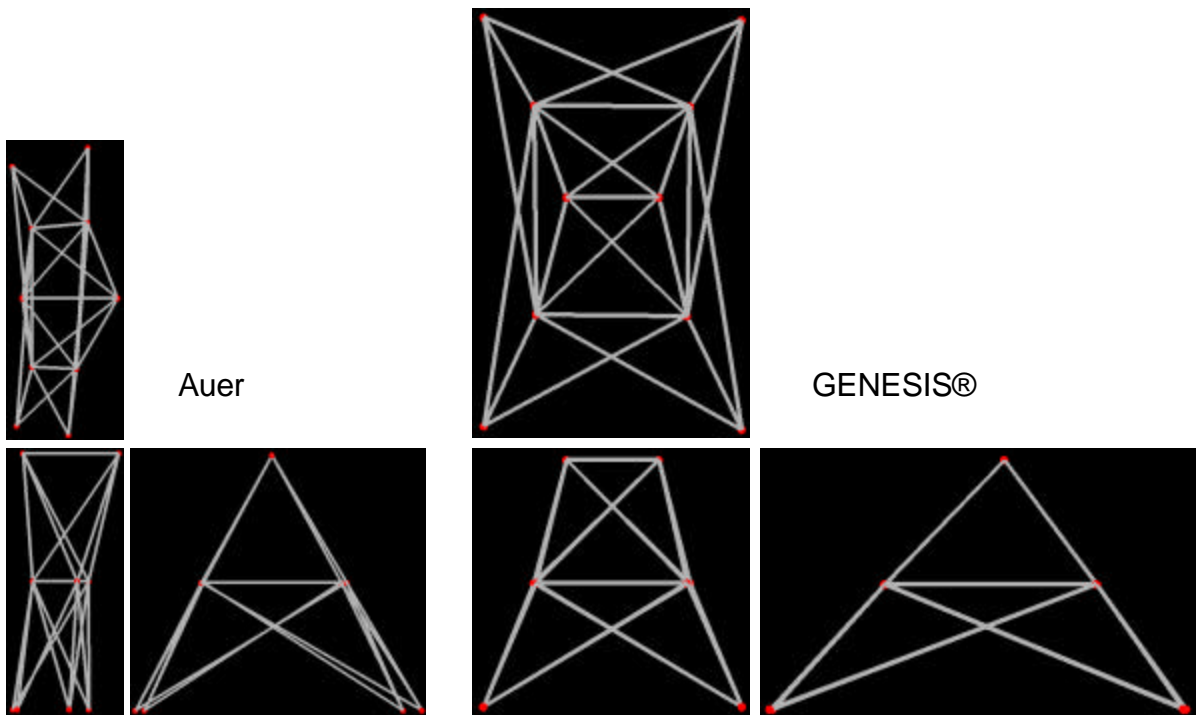


Figure 4.2.3. Shape solutions to study 4

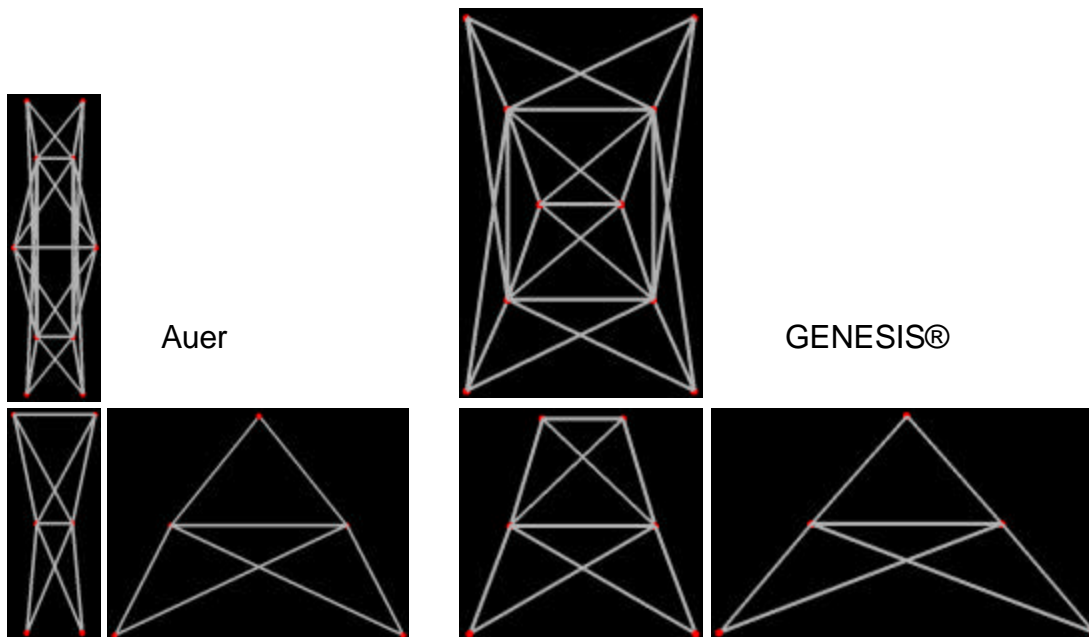


Figure 4.2.4. Shape solutions to study 5

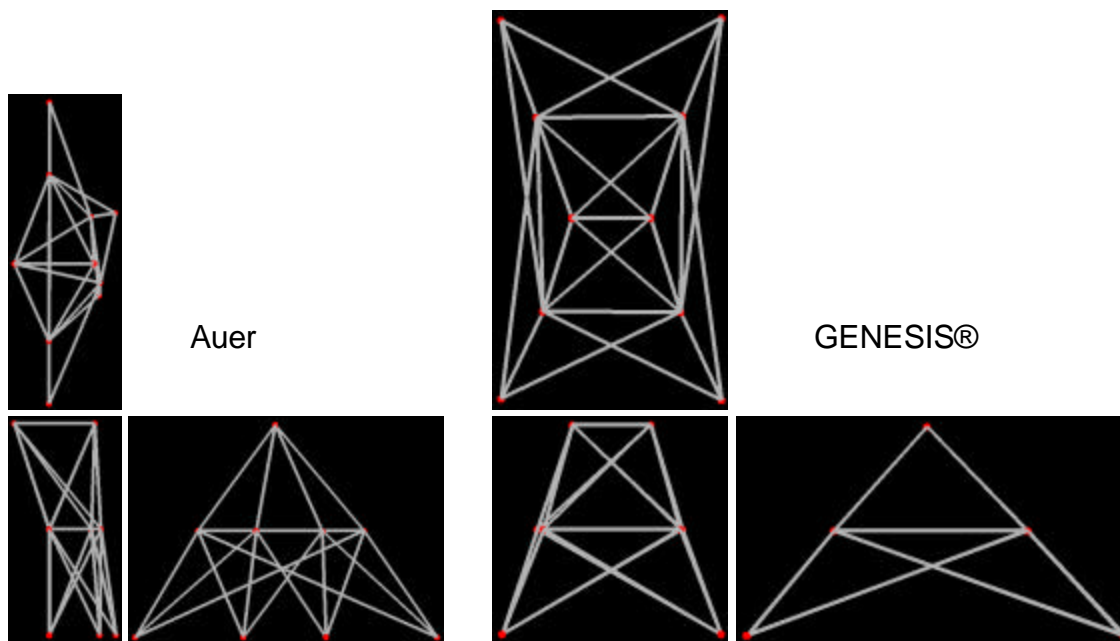


Figure 4.2.5. Shape solutions to study 6

Several points can be made by viewing the figures above. The solutions presented by GENESIS® have a similar shape to each other, with a very broad footprint from the top and a wide stance from both sides. It is also notable the amount of symmetry produced,

even when shape symmetry was not used. The solutions produced by the genetic algorithm have radically different shapes than those produced by GENESIS®. The solution to study 3 has a shape similar to the GENESIS® solution, but the footprint is much smaller. In fact, all four genetic algorithm solutions produced structures with much smaller footprints than those produced by GENESIS®. The strength to the genetic algorithm solutions is that they present the designer with many unique options while maintaining a competitive mass.

4.2.2 Impact of Design Variable Reduction

In addition to the structural mass and fitness, the number of variables and number of evaluations for each study were recorded. These extra values can lead to insights about the effects of design variable reduction in the genetic algorithm. It is hypothesized that the reduction of design variables through the use of correct symmetries can lead to better solutions with fewer fitness evaluations. Table 4.2.8 shows the number of design variables, the fitness value, the number of evaluations for each of the six studies performed, and a score defined by (4.2.1). The table shows the results from only one set of experiments because the intent of this study is not to show statistical significance, but to present future research options.

| Study | 1 | 2 | 3 | 4 | 5 | 6 |
|-------------|---------|---------|--------|--------|--------|--------|
| Variables | 8 | 25 | 12 | 24 | 29 | 41 |
| Fitness | 160.009 | 163.295 | 60.891 | 58.530 | 67.646 | 56.171 |
| Evaluations | 44370 | 65496 | 51546 | 112718 | 293780 | 181930 |
| SCORE | 160.0 | 171.1 | 60.9 | 65.5 | 99.4 | 70.4 |

Table 4.2.8. Results for design variable reduction experiment

Equation (4.2.1) resembles the fitness equation used in the genetic algorithm. It uses the minimum number of evaluations found by any solution as a limit for punishment. This means that all but one solution will have a score higher than their fitness value.

Punishment is much lighter in this equation, making the number of evaluations have a 10% effect on the score and the fitness a 90% effect.

$$SCORE = FITNESS \cdot \left(1 + 0.1 \left(\frac{EVALUATION S}{\min(EVALUATION S)} - 1 \right) \right) \quad (4.2.1)$$

The studies in Table 4.2.8 are divided into two cases: solutions exploring size optimization only and solutions exploring size and shape optimization. It is easy to see that in the first case the reduction of design variables through symmetry produced a lower fitness value in fewer evaluations. This case fully supports the hypothesis stated.

In the case of size and shape optimization, it is expected that as the number of variables increases, so should the score. This is the case for studies 3, 4, and 5, but study 6 presents an odd data point. The fitness in study 6 actually drops to the lowest value of all the studies, while at the same time lowering the number of evaluations from the study 5.

This experiment is difficult to draw concrete conclusions because the nature of applying variable symmetries is almost an art. The designer must have a good sense of when and how to apply symmetries in order for feasible results to be found. It is clear from Table 4.2.8 that the combination of shape and size variable reduction leads to much faster results while still producing comparable fitness values for this problem. Much of the application of variable symmetries is dependent on the structure's initial shape, size, loading conditions, and constraints. Some structures thrive on the use of symmetry, while others will produce such poor results that the time saved is meaningless. This structure lends itself to the use of size and shape symmetries.

5.0 CASE STUDY

The Society of Automotive Engineers (SAE) hosts an annual collegiate competition called Formula SAE®. The competition is for SAE student members to conceive, design, fabricate, and compete with small formula-style racing cars. The cars are built with a team effort over a period of one to two years and are taken to the annual competition for judging and comparison with up to 140 other vehicles from colleges and universities throughout the world.

The vehicle's frame is the largest and second heaviest (next to the engine) single component on the Formula SAE® car. It acts as the central mounting bracket for systems and components such as: suspension, engine, drive train, driver controls, the driver, and the body. The frame must maintain a high strength, low weight, and appealing aesthetics while providing usable mounting points for all necessary components in their desired locations. Many SAE rules also specify certain tubing sizes, shapes, and locations for the frame due to safety concerns.

The optimization problem presented is the minimization of mass for a Formula SAE® frame through size and shape optimization. Figure 5.0.1 shows a solid model of the University of Idaho's 2005 frame. The theoretical weight of the structure is 59.4 lb according to SolidWorks® mass calculations. The basic dimensions are 36 inches tall, 92 inches long, 26 inches wide, and a minimum ground clearance of 1 inch with the suspension in full compression. The frame is constructed from round and square aircraft grade 4130 steel thin-walled tubing with a post-weld yield strength of 62500 psi (a factor of safety of 1.2 has been applied). Square tubes are 1 x 1 x 0.065 inches and round tubes

range in outer diameter from 1 inch to 0.5 inches with wall thicknesses ranging from 0.095 inches to 0.035 inches. C-channel and 0.125 inch plate is also used for creating brackets and tabs needed for mounting components.

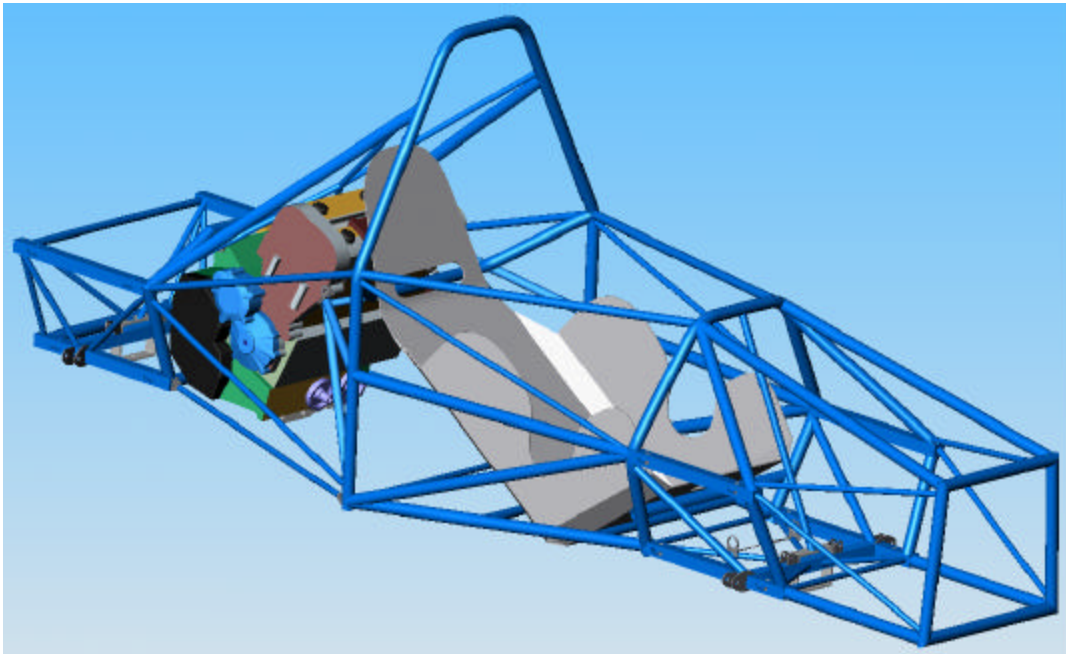


Figure 5.0.1. Solid model of 2005 FSAE frame

The model was simplified by excluding bends and modeling the seat and engine using frame members. These components are used in the analysis because they provide rigidity to the frame and they are components that are always mounted when the car is in use. They are modeled through simplified frame elements with zero mass and infinite yield strength so they do not affect the fitness value of the frame. The initial weight of the frame is 57.956 lb. Appendix C holds the information used in modeling the initial structure. In addition to the seat and engine, two sections have been included: the suspension and the soft-constraint base stand. The suspension linkages (a-arms, pull rods, shocks, and bell cranks) are modeled using truss elements to mimic their load carrying capabilities. The

uprights connect the suspension linkages to the ground and are modeled using frame elements.

A difficulty arises when modeling the frame: it is a dynamic system for which static modeling techniques do not apply. The structure below the frame is used to constrain the system while allowing the frame to rigidly rotate and translate, which does not cause element stresses to occur in the frame. The entire model is constrained only at the four lowest points of the support structure and acts as a constraint-distribution device. This structure is modeled using small truss elements made from a material exhibiting the linear elastic properties of lead. It allows the model constraints to be fully satisfied while preventing non-realistic stress concentrations from occurring in the frame.

Five loading cases are used in the analysis of the frame and are shown in Table 5.0.1. These cases are representative of major categories of extreme situations, and they are based on the car's weight, center of gravity, and tire friction limits. X and Y forces are applied at the bottom of the uprights where they contact the ground, while the Z forces and X moments (M_x) are applied at the center of the uprights. The Z force is not applied at the ground so it does not induce false moments, and the X moments are applied at the upright center to mimic braking forces. Engine forces are only applied under acceleration loading conditions, and are located at the output shaft of the engine. Because the loads represent only one direction of turning, all frame nodes are symmetric about the yz coordinate plane. Tubing properties are also symmetric across this plane to ensure a fully symmetric structure under non-symmetric loading.

| Force Matrix (lb), (in*lb) | Front | | | | | | | | Rear | | | | | | | |
|-------------------------------|-------|-----|------|------|-------|-----|------|------|------|-----|-----|-----|-------|-----|------|------|
| | Left | | | | Right | | | | Left | | | | Right | | | |
| | X | Y | Z | Mx | X | Y | Z | Mx | X | Y | Z | Mx | X | Y | Z | Mx |
| Bump+Acc * | 0 | 110 | 0 | 0 | 0 | 110 | 0 | 0 | 0 | 590 | 413 | 0 | 0 | 590 | 413 | 0 |
| Bump+Brake | 0 | 562 | -393 | 3930 | 0 | 562 | -393 | 3930 | 0 | 138 | -96 | 960 | 0 | 138 | -96 | 960 |
| Bump+Turn | 34 | 48 | 0 | 0 | 436 | 624 | 0 | 0 | 36 | 52 | 0 | 0 | 473 | 676 | 0 | 0 |
| Turn+Acc+Bump * | 0 | 31 | 0 | 0 | 190 | 380 | 0 | 0 | 151 | 303 | 151 | 0 | 374 | 748 | 374 | 0 |
| Turn+Brake+Bump | 146 | 291 | -146 | 1460 | 351 | 703 | -351 | 3510 | 0 | 20 | 0 | 100 | 213 | 425 | -213 | 2130 |

* Apply engine forces to these models

| Engine Forces | X | Y | Z | Mx |
|---------------|---|------|-----|-------|
| | 0 | -410 | 320 | -1040 |

Table 5.0.1. Input Force model for FSAE car

Four studies were performed on the frame: size optimization, shape optimization, size and shape optimization, and a size and shape optimization with a uniform starting point. The first three studies have been described in previous content and all cases utilize nodal and elemental symmetries. The tube sizes in these studies begin at the current design size, meaning it has already been optimized by hand. The last study uses a starting point of 1 x 0.035 inch tubing for all members not restricted by the rules. This study was performed to verify that the solutions to the previous studies were not influenced by the initial design. All four studies were performed three times each and arrived at identical solutions within the study all three times.

5.1 Results

The final solution to each study is shown in Table 5.1.1. With an initial fitness and mass of 57.956 lb, it is clear that the size optimization was more useful in reducing mass. Where size and shape optimization were applied (studies 3 and 4) very minimal gains were found over size-only optimization. Study 4 proves that studies 1 through 3 were not significantly influenced by the initial design, because it arrived at the same solution as study 3 from a different starting point. The torsional rigidity is also shown for each study, which is a measure of torsional frame stiffness along its length. This measure dropped by

over 200 ft*lb/deg in studies 1, 3, and 4. These studies also happen to be cases where size optimization has been applied. A decrease in mass of less than 2 lb that causes a decrease in rigidity of over 200 ft*lb/deg is not satisfactory.

| Study | Original | 1 | 2 | 3 | 4 |
|----------------|----------|----------|----------|----------|----------|
| Fitness | 57.956 | 56.2525 | 57.8672 | 56.1713 | 56.1713 |
| Mass (lb) | 57.956 | 56.2525 | 57.8672 | 56.1713 | 56.1713 |
| TR (ft*lb/deg) | 1782.824 | 1561.808 | 1781.845 | 1558.692 | 1558.692 |

Table 5.1.1. Final results for the miniature race car frame

It may seem beneficial to include the maximization of torsional rigidity in the objective of the optimization study, but the software does not directly support this. The torsional rigidity measure requires a different set of constraints from the model used, and the software only supports one constraint set. Bounding deflections on the frame is also unusable because the soft constraint structure allows the frame to rigidly translate and rotate far past any reasonable bounds. The final reason for not including the torsional rigidity in the optimization is due to the fact that the measure is comprised of four deflection values, three nodal location values, and one load value. The angle of twist between the front and rear of the frame must be calculated using the deflection results and nodal locations. This is then combined with the equivalent moment due to a vertical force placed on one wheel of the car. The torsional rigidity is a measure specific to this design problem and should be handled with engineering judgment rather than optimization software. The optimization software is intended to handle a large number of problems and their most common objectives and boundaries, not a single problem and its specific objectives.

5.2 Implications

The conclusion drawn is that the original structure is the best solution. This is due to several factors: a small decrease in mass, a large decrease in rigidity, and a small change in the shape of the structure. This particular problem does not serve to benefit directly from the use of the genetic algorithm software, but the results of study 4 imply general usefulness for frame optimization.

Study 4 was used to emulate the design process used when the structure was initially optimized by hand. The original process took nearly one week to set up and solve with two student engineers working full days on the problem. It was a process that involved analyzing the structure for all five loading cases and documenting the maximum stress in each member and taking symmetry into consideration. Tubes were then downsized or upsized based on stress limits set, and the process was continued until no changes could be made. Study 4 began with the same starting point as the hand optimization, but took only 20 minutes to come to the same solution. This means that designers can begin with a certain layout, basic shape, and roughly sized frame and optimize in a much shorter time than before possible.

Frame optimization requires much care and consideration when setting size and shape limits. Usually, stress and mass are not the only governing factors when dealing with frame optimization. This problem must also face rules and other physical constraints placed on either node locations and tubing sizes. If the problem is carefully set-up and all external constraints are kept in mind, the results of the frame optimization can be very realistic and usable.

6.0 CONCLUSIONS AND RECOMMENDATIONS

The genetic algorithm developed for structural optimization is a usable tool in the design and optimization of truss and frame structures. It is competitive with commercial optimization software while presenting a wider variety of possible solutions. Features such as nodal and elemental symmetries, user defined optimization parameters, and visualization utilities make the software package easily adaptable to most structural optimization problems. The software is best used as a design guide rather than a final solution to structural problems. It cannot replace engineering judgment and experience, but it can give valuable insight to problems.

Future work for this genetic algorithm software package might include additions to the visualization software to accommodate stress and displacement results, the allowance of creating structures from the visualization software rather than text files, symmetry recognition within the genetic algorithm, and topological optimization capabilities as described by Kawamura (2002) and Azid (2002). The addition of topology optimization would complete the software so that it covers all major areas of structural optimization. Better error and warning output would also be of great importance, especially when new users are introduced to the software.

The recommendation for the University of Idaho's Formula SAE® frame is to find a different topology design. The project rules specify that significant changes must be made to the car frame between years of competition. The results found in this work suggest that no significant changes can be made using only size and shape variation.

7.0 REFERENCES

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8.0 APPENDIX A: MODEL DATA FOR THE 10-BAR TRUSS

Input files for the initial structure:

| NODE | X-POS | Y-POS | Z-POS | X-CONS | Y-CONS | Z-CONS | Rx-CONS | Ry-CONS | Rz-CONS |
|------|-------|-------|-------|--------|--------|--------|---------|---------|---------|
| 1 | 720 | 360 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2 | 720 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 3 | 360 | 360 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 4 | 360 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 360 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

| ELEMENT | TYPE | NODE1 | NODE2 | NODE_K | MATL | PROPS |
|---------|-----------|-------|-------|--------|------|-------------------|
| | (1) TRUSS | | | | | AREA |
| | (2) BEAM | | | | | SEC TYPE GEOMETRY |
| 1 | 1 | 3 | 5 | | 1 | 10 |
| 2 | 1 | 1 | 3 | | 1 | 10 |
| 3 | 1 | 4 | 6 | | 1 | 10 |
| 4 | 1 | 2 | 4 | | 1 | 10 |
| 5 | 1 | 3 | 4 | | 1 | 10 |
| 6 | 1 | 1 | 2 | | 1 | 10 |
| 7 | 1 | 4 | 5 | | 1 | 10 |
| 8 | 1 | 3 | 6 | | 1 | 10 |
| 9 | 1 | 2 | 3 | | 1 | 10 |
| 10 | 1 | 1 | 4 | | 1 | 10 |

| MATL | NAME | E | ν | YIELD | DENSITY |
|------|----------|-----------|-------|-------|---------|
| 1 | Aluminum | 100000000 | 0.334 | 25000 | 0.1 |

| LOAD | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM |
|------|------|-------|---------|-------|-------|-------|-------|
| 1 | 2 | 0 | -100000 | 0 | 0 | 0 | 0 |
| 2 | 4 | 0 | -100000 | 0 | 0 | 0 | 0 |

Optimization file for the initial structure of study 1:

NODAL BOUNDARIES

NODE # X-LO X-HI Y-LO Y-HI Z-LO Z-HI

NODAL SYMMETRIES

NODE # WITH # ABOUT PLANE (1)XZ (2)YZ (3)XY

DISPLACEMENT BOUNDARIES

NODE # DX DY DZ RX RY RZ

TRUSS ELEMENT BOUNDARIES

TRUSS # A-LO A-HI

| | | |
|----|-----|----|
| 1 | 0.1 | 10 |
| 2 | 0.1 | 10 |
| 3 | 0.1 | 10 |
| 4 | 0.1 | 10 |
| 5 | 0.1 | 10 |
| 6 | 0.1 | 10 |
| 7 | 0.1 | 10 |
| 8 | 0.1 | 10 |
| 9 | 0.1 | 10 |
| 10 | 0.1 | 10 |

TRUSS ELEMENT EQUALITIES

TRUSS # WITH #

ALL POSSIBLE CROSS SECTIONS

SEC # TYPE GEOMETRY

CROSS SECTION SETS

SET # SECTION #

BEAM ELEMENT BOUNDARIES

BEAM # SET #

Intermediate result for study 1:

| ELEMENT | AREA |
|---------|----------|
| 1 | 7.93286 |
| 2 | 0.100553 |
| 3 | 8.07296 |
| 4 | 3.93612 |
| 5 | 0.1 |
| 6 | 0.1 |
| 7 | 5.75408 |
| 8 | 5.56091 |
| 9 | 5.56317 |
| 10 | 0.116271 |

| RESULTS | |
|----------|---------|
| FITNESS: | 1593.94 |
| MASS: | 1593.94 |
| GENS: | 2658 |
| TIME: | 201 |

Final Result for study 1:

| ELEMENT | AREA |
|---------|----------|
| 1 | 7.9324 |
| 2 | 0.1 |
| 3 | 8.06782 |
| 4 | 3.93227 |
| 5 | 0.1 |
| 6 | 0.1 |
| 7 | 5.75271 |
| 8 | 5.56106 |
| 9 | 5.56107 |
| 10 | 0.116429 |

| RESULTS | |
|----------|---------|
| FITNESS: | 1593.42 |
| MASS: | 1593.42 |
| GENS: | 1752 |
| TIME: | 182 |

Optimization file for initial structure of study 2:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
| 1 | 501 | 720 | 50 | 360 | | |
| 3 | 300 | 600 | 200 | 400 | | |
| 5 | | | 360 | 720 | | |

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE | (1)XZ | (2)YZ | (3)XY |
|--------|--------|-------------|-------|-------|-------|
|--------|--------|-------------|-------|-------|-------|

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
|--------|----|----|----|----|----|----|

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.1 | 10 |
| 2 | 0.1 | 10 |
| 3 | 0.1 | 10 |
| 4 | 0.1 | 10 |
| 5 | 0.1 | 10 |
| 6 | 0.1 | 10 |
| 7 | 0.1 | 10 |
| 8 | 0.1 | 10 |
| 9 | 0.1 | 10 |
| 10 | 0.1 | 10 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
|---------|--------|

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

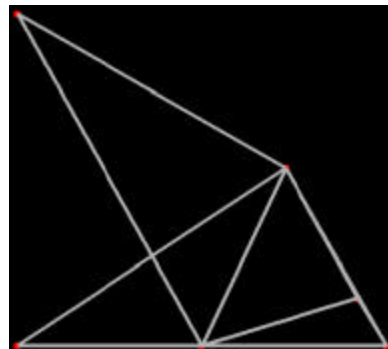
| BEAM # | SET # |
|--------|-------|
|--------|-------|

Intermediate result for study 2:

| ELEMENT | AREA |
|---------|---------|
| 1 | 4.88468 |
| 2 | 0.1 |
| 3 | 4.16926 |
| 4 | 2.1186 |
| 5 | 0.1 |
| 6 | 0.1 |
| 7 | 4.48289 |
| 8 | 2.52501 |
| 9 | 4.44386 |
| 10 | 0.1 |

| NODE | X-POS | Y-POS |
|------|---------|---------|
| 1 | 664.553 | 99.175 |
| 2 | 720 | 0 |
| 3 | 523.412 | 371.95 |
| 4 | 360 | 0 |
| 5 | 0 | 694.251 |
| 6 | 0 | 0 |

| RESULTS | |
|----------|---------|
| FITNESS: | 1237.76 |
| MASS: | 1237.76 |
| GENS: | 2263 |
| TIME: | 141 |

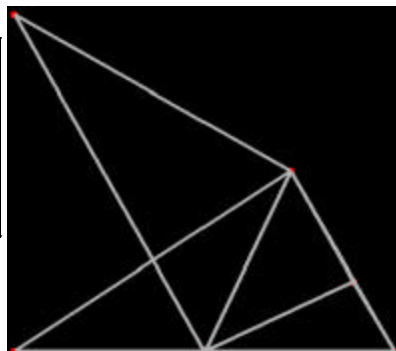


Final result for study 2:

| ELEMENT | AREA |
|---------|---------|
| 1 | 4.88465 |
| 2 | 0.1 |
| 3 | 4.16913 |
| 4 | 2.11574 |
| 5 | 0.1 |
| 6 | 0.1 |
| 7 | 4.48253 |
| 8 | 2.51928 |
| 9 | 4.42643 |
| 10 | 0.1 |

| NODE | X-POS | Y-POS |
|------|---------|---------|
| 1 | 642.427 | 143.557 |
| 2 | 720 | 0 |
| 3 | 523.361 | 371.946 |
| 4 | 360 | 0 |
| 5 | 0 | 694.251 |
| 6 | 0 | 0 |

| RESULTS | |
|----------|---------|
| FITNESS: | 1236.46 |
| MASS: | 1236.46 |
| GENS: | 2134 |
| TIME: | 127 |



9.0 APPENDIX B: MODEL DATA FOR THE 25-BAR TRUSS

Input files for the initial structure:

| NODE | X-POS | Y-POS | Z-POS | X-CONS | Y-CONS | Z-CONS | Rx-CONS | Ry-CONS | Rz-CONS |
|------|-------|-------|-------|--------|--------|--------|---------|---------|---------|
| 1 | -37.5 | 0 | 200 | 0 | 0 | 0 | 1 | 1 | 1 |
| 2 | 37.5 | 0 | 200 | 0 | 0 | 0 | 1 | 1 | 1 |
| 3 | -37.5 | 37.5 | 100 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 37.5 | 37.5 | 100 | 0 | 0 | 0 | 1 | 1 | 1 |
| 5 | 37.5 | -37.5 | 100 | 0 | 0 | 0 | 1 | 1 | 1 |
| 6 | -37.5 | -37.5 | 100 | 0 | 0 | 0 | 1 | 1 | 1 |
| 7 | -100 | 100 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | 100 | 100 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 100 | -100 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 10 | -100 | -100 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

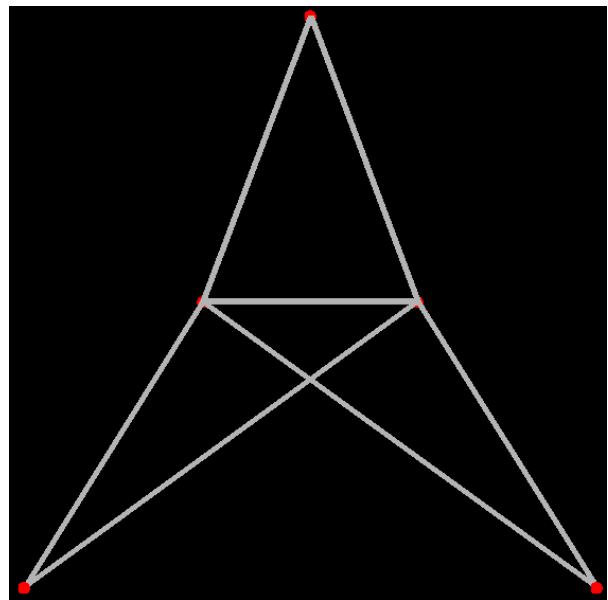
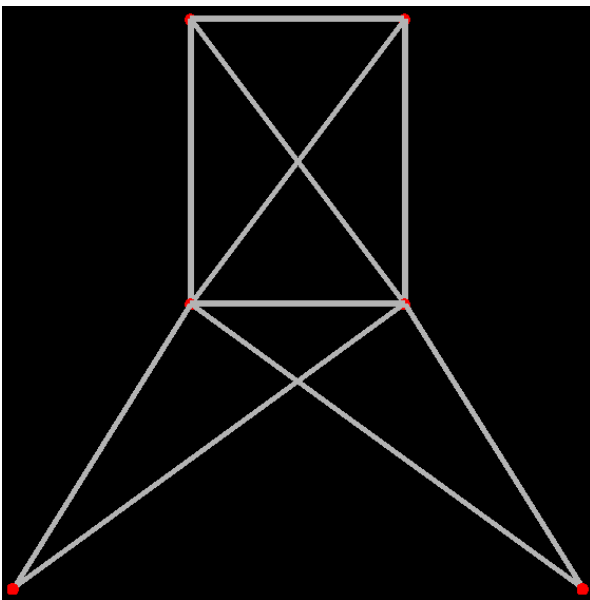
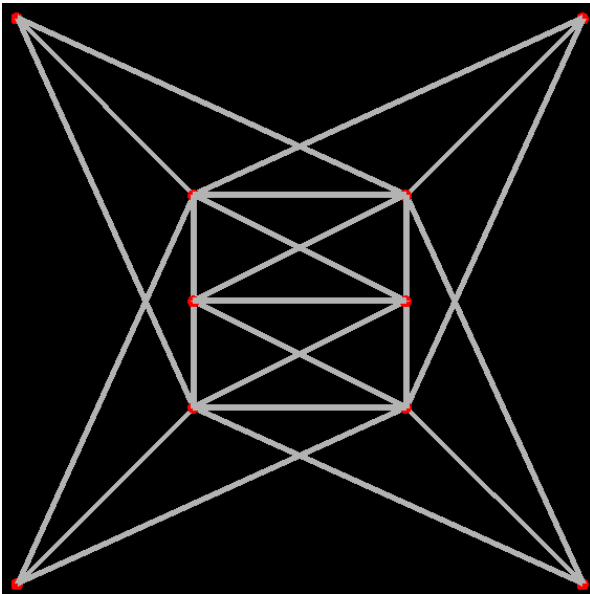
| ELEMENT | TYPE | NODE1 | NODE2 | NODE_K | MATL | PROPS |
|---------|-----------|-------|-------|--------|------|-------------------|
| | (1) TRUSS | | | | | AREA |
| | (2) BEAM | | | | | SEC TYPE GEOMETRY |
| 1 | 1 | 1 | 2 | | 1 | 3 |
| 2 | 1 | 1 | 4 | | 1 | 3 |
| 3 | 1 | 2 | 3 | | 1 | 3 |
| 4 | 1 | 1 | 5 | | 1 | 3 |
| 5 | 1 | 2 | 6 | | 1 | 3 |
| 6 | 1 | 2 | 4 | | 1 | 3 |
| 7 | 1 | 2 | 5 | | 1 | 3 |
| 8 | 1 | 1 | 3 | | 1 | 3 |
| 9 | 1 | 1 | 6 | | 1 | 3 |
| 10 | 1 | 3 | 6 | | 1 | 3 |
| 11 | 1 | 4 | 5 | | 1 | 3 |
| 12 | 1 | 3 | 4 | | 1 | 3 |
| 13 | 1 | 5 | 6 | | 1 | 3 |
| 14 | 1 | 3 | 10 | | 1 | 3 |
| 15 | 1 | 6 | 7 | | 1 | 3 |
| 16 | 1 | 4 | 9 | | 1 | 3 |
| 17 | 1 | 5 | 8 | | 1 | 3 |
| 18 | 1 | 4 | 7 | | 1 | 3 |
| 19 | 1 | 3 | 8 | | 1 | 3 |
| 20 | 1 | 5 | 10 | | 1 | 3 |
| 21 | 1 | 6 | 9 | | 1 | 3 |
| 22 | 1 | 6 | 10 | | 1 | 3 |
| 23 | 1 | 3 | 7 | | 1 | 3 |
| 24 | 1 | 4 | 8 | | 1 | 3 |
| 25 | 1 | 5 | 9 | | 1 | 3 |

| MATL | NAME | E | v | YIELD | DENSITY |
|------|----------|----------|-------|-------|---------|
| 1 | Aluminum | 10000000 | 0.334 | 35294 | 0.1 |

2

| LOAD | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM |
|------|------|-------|-------|-------|-------|-------|-------|
| 1 | 1 | 0 | 20000 | -5000 | 0 | 0 | 0 |
| 2 | 2 | 0 | 20000 | -5000 | 0 | 0 | 0 |

| LOAD | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM |
|------|------|-------|-------|-------|-------|-------|-------|
| 1 | 1 | 1000 | 10000 | -5000 | 0 | 0 | 0 |
| 2 | 2 | 0 | 10000 | -5000 | 0 | 0 | 0 |
| 3 | 3 | 500 | 0 | 0 | 0 | 0 | 0 |
| 4 | 6 | 500 | 0 | 0 | 0 | 0 | 0 |



Optimization file for initial structure of study 1:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
|--------|------|------|------|------|------|------|

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE | (1)XZ | (2)YZ | (3)XY |
|--------|--------|-------------|-------|-------|-------|
|--------|--------|-------------|-------|-------|-------|

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 22 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
| 3 | 2 |
| 4 | 2 |
| 5 | 2 |
| 7 | 6 |
| 8 | 6 |
| 9 | 6 |
| 11 | 10 |
| 13 | 12 |
| 15 | 14 |
| 16 | 14 |
| 17 | 14 |
| 19 | 18 |
| 20 | 18 |
| 21 | 18 |
| 23 | 22 |
| 24 | 22 |
| 25 | 22 |

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

| BEAM # | SET # |
|--------|-------|
|--------|-------|

BEAM ELEMENT EQUALITIES

| BEAM # | WITH # |
|--------|--------|
|--------|--------|

Intermediate result for study 1:

| ELEMENT | AREA | RESULTS |
|---------|-----------|------------------|
| 1 | 0.01 | FITNESS: 162.282 |
| 2 | 0.0258538 | MASS: 162.282 |
| 3 | 0.0258538 | GENS: 1484 |
| 4 | 0.0258538 | TIME: 226 |
| 5 | 0.0258538 | |
| 6 | 1.30162 | |
| 7 | 1.30162 | |
| 8 | 1.30162 | |
| 9 | 1.30162 | |
| 10 | 0.01 | |
| 11 | 0.01 | |
| 12 | 0.709158 | |
| 13 | 0.709158 | |
| 14 | 0.360856 | |
| 15 | 0.360856 | |
| 16 | 0.360856 | |
| 17 | 0.360856 | |
| 18 | 0.01 | |
| 19 | 0.01 | |
| 20 | 0.01 | |
| 21 | 0.01 | |
| 22 | 1.26615 | |
| 23 | 1.26615 | |
| 24 | 1.26615 | |
| 25 | 1.26615 | |

Final result for study 1:

| ELEMENT | AREA | RESULTS |
|---------|-----------|------------------|
| 1 | 0.01 | FITNESS: 160.009 |
| 2 | 0.0245825 | MASS: 160.009 |
| 3 | 0.0245825 | GENS: 2257 |
| 4 | 0.0245825 | TIME: 349 |
| 5 | 0.0245825 | |
| 6 | 1.27249 | |
| 7 | 1.27249 | |
| 8 | 1.27249 | |
| 9 | 1.27249 | |
| 10 | 0.01 | |
| 11 | 0.01 | |
| 12 | 0.773481 | |
| 13 | 0.773481 | |
| 14 | 0.266582 | |
| 15 | 0.266582 | |
| 16 | 0.266582 | |
| 17 | 0.266582 | |
| 18 | 0.01 | |
| 19 | 0.01 | |
| 20 | 0.01 | |
| 21 | 0.01 | |
| 22 | 1.358 | |
| 23 | 1.358 | |
| 24 | 1.358 | |
| 25 | 1.358 | |

Optimization file for initial structure of study 2:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
|--------|------|------|------|------|------|------|

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE | (1)XZ | (2)YZ | (3)XY |
|--------|--------|-------------|-------|-------|-------|
|--------|--------|-------------|-------|-------|-------|

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 3 | 0.01 | 3 |
| 4 | 0.01 | 3 |
| 5 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 7 | 0.01 | 3 |
| 8 | 0.01 | 3 |
| 9 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 11 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 13 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 15 | 0.01 | 3 |
| 16 | 0.01 | 3 |
| 17 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 19 | 0.01 | 3 |
| 20 | 0.01 | 3 |
| 21 | 0.01 | 3 |
| 22 | 0.01 | 3 |
| 23 | 0.01 | 3 |
| 24 | 0.01 | 3 |
| 25 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
|---------|--------|

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

| BEAM # | SET # |
|--------|-------|
|--------|-------|

BEAM ELEMENT EQUALITIES

| BEAM # | WITH # |
|--------|--------|
|--------|--------|

Intermediate result for study 2:

| ELEMENT | AREA | RESULTS |
|---------|-----------|------------------|
| 1 | 0.01 | FITNESS: 172.969 |
| 2 | 0.170945 | MASS: 172.969 |
| 3 | 0.226078 | GENS: 2930 |
| 4 | 0.209963 | TIME: 374 |
| 5 | 0.125062 | |
| 6 | 1.65557 | |
| 7 | 1.10639 | |
| 8 | 1.6278 | |
| 9 | 1.03938 | |
| 10 | 0.01 | |
| 11 | 0.382877 | |
| 12 | 0.751072 | |
| 13 | 1.06784 | |
| 14 | 0.29321 | |
| 15 | 0.326736 | |
| 16 | 0.446523 | |
| 17 | 0.01 | |
| 18 | 0.0167361 | |
| 19 | 0.010389 | |
| 20 | 0.01 | |
| 21 | 0.01 | |
| 22 | 0.987113 | |
| 23 | 1.57225 | |
| 24 | 1.09323 | |
| 25 | 1.47362 | |

Finals result for study 2:

| ELEMENT | AREA | RESULTS |
|---------|-----------|------------------|
| 1 | 0.01 | FITNESS: 163.295 |
| 2 | 0.0112743 | MASS: 163.295 |
| 3 | 0.03 | GENS: 1730 |
| 4 | 0.0629406 | TIME: 225 |
| 5 | 0.01 | |
| 6 | 1.46 | |
| 7 | 1.1263 | |
| 8 | 1.43 | |
| 9 | 1.2213 | |
| 10 | 0.01 | |
| 11 | 0.18 | |
| 12 | 0.697143 | |
| 13 | 0.87 | |
| 14 | 0.28142 | |
| 15 | 0.279631 | |
| 16 | 0.340144 | |
| 17 | 0.202011 | |
| 18 | 0.083944 | |
| 19 | 0.0628795 | |
| 20 | 0.01 | |
| 21 | 0.01 | |
| 22 | 1.19 | |
| 23 | 1.37001 | |
| 24 | 1.28455 | |
| 25 | 1.3709 | |

Optimization file for initial structure of study 3:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
| 4 | 5 | 100 | 5 | 100 | | |
| 8 | 5 | 200 | 5 | 200 | | |

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE (1)XZ (2)YZ (3)XY |
|--------|--------|-------------------------------|
| 3 | 4 | 2 |
| 5 | 4 | 1 |
| 6 | 3 | 1 |
| 7 | 8 | 2 |
| 9 | 8 | 1 |
| 10 | 7 | 1 |

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 22 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
| 3 | 2 |
| 4 | 2 |
| 5 | 2 |
| 7 | 6 |
| 8 | 6 |
| 9 | 6 |
| 11 | 10 |
| 13 | 12 |
| 15 | 14 |
| 16 | 14 |
| 17 | 14 |
| 19 | 18 |
| 20 | 18 |
| 21 | 18 |
| 23 | 22 |
| 24 | 22 |
| 25 | 22 |

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

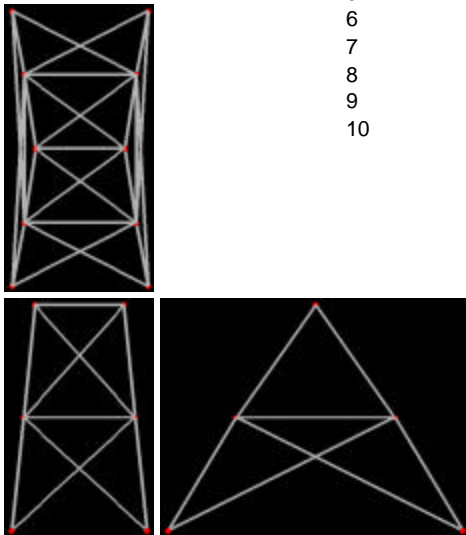
| BEAM # | SET # |
|--------|-------|
|--------|-------|

BEAM ELEMENT EQUALITIES

| BEAM # | WITH # |
|--------|--------|
|--------|--------|

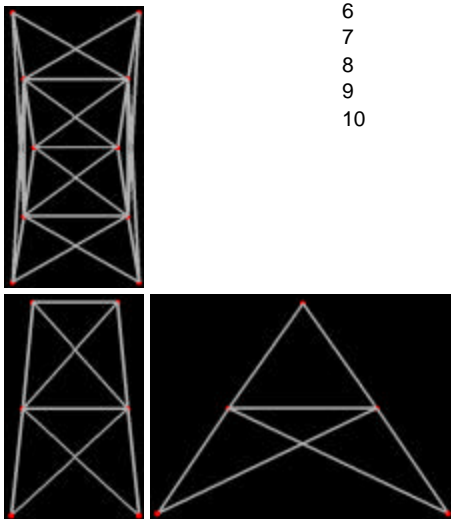
Intermediate result for study 3:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS |
|---------|-----------|------------------|------|----------|----------|-------|
| 1 | 0.0426992 | FITNESS: 62.8453 | 1 | -37.5 | 0 | 200 |
| 2 | 0.0202789 | MASS: 62.8453 | 2 | 37.5 | 0 | 200 |
| 3 | 0.0202789 | GENS: 2834 | 3 | -46.5918 | 65.1964 | 100 |
| 4 | 0.0202789 | TIME: 302 | 4 | 46.5918 | 65.1964 | 100 |
| 5 | 0.0202789 | | 5 | 46.5918 | -65.1964 | 100 |
| 6 | 0.596053 | | 6 | -46.5918 | -65.1964 | 100 |
| 7 | 0.596053 | | 7 | -56.6229 | 120.774 | 0 |
| 8 | 0.596053 | | 8 | 56.6229 | 120.774 | 0 |
| 9 | 0.596053 | | 9 | 56.6229 | -120.774 | 0 |
| 10 | 0.01 | | 10 | -56.6229 | -120.774 | 0 |
| 11 | 0.01 | | | | | |
| 12 | 0.01 | | | | | |
| 13 | 0.01 | | | | | |
| 14 | 0.0400258 | | | | | |
| 15 | 0.0400258 | | | | | |
| 16 | 0.0400258 | | | | | |
| 17 | 0.0400258 | | | | | |
| 18 | 0.0267141 | | | | | |
| 19 | 0.0267141 | | | | | |
| 20 | 0.0267141 | | | | | |
| 21 | 0.0267141 | | | | | |
| 22 | 0.594678 | | | | | |
| 23 | 0.594678 | | | | | |
| 24 | 0.594678 | | | | | |
| 25 | 0.594678 | | | | | |



Final result for study 3:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS |
|---------|-----------|------------------|------|----------|----------|-------|
| 1 | 0.0238683 | FITNESS: 60.8913 | 1 | -37.5 | 0 | 200 |
| 2 | 0.0220379 | MASS: 60.8913 | 2 | 37.5 | 0 | 200 |
| 3 | 0.0220379 | GENS: 2295 | 3 | -46.5165 | 65.9359 | 100 |
| 4 | 0.0220379 | TIME: 227 | 4 | 46.5165 | 65.9359 | 100 |
| 5 | 0.0220379 | | 5 | 46.5165 | -65.9359 | 100 |
| 6 | 0.59057 | | 6 | -46.5165 | -65.9359 | 100 |
| 7 | 0.59057 | | 7 | -56.4627 | 129.183 | 0 |
| 8 | 0.59057 | | 8 | 56.4627 | 129.183 | 0 |
| 9 | 0.59057 | | 9 | 56.4627 | -129.183 | 0 |
| 10 | 0.01 | | 10 | -56.4627 | -129.183 | 0 |
| 11 | 0.01 | | | | | |
| 12 | 0.01 | | | | | |
| 13 | 0.01 | | | | | |
| 14 | 0.01 | | | | | |
| 15 | 0.01 | | | | | |
| 16 | 0.01 | | | | | |
| 17 | 0.01 | | | | | |
| 18 | 0.0251626 | | | | | |
| 19 | 0.0251626 | | | | | |
| 20 | 0.0251626 | | | | | |
| 21 | 0.0251626 | | | | | |
| 22 | 0.592479 | | | | | |
| 23 | 0.592479 | | | | | |
| 24 | 0.592479 | | | | | |
| 25 | 0.592479 | | | | | |



Optimization file for initial structure of study 4:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
| 3 | -100 | -5 | 5 | 100 | | |
| 4 | 5 | 100 | 5 | 100 | | |
| 5 | 5 | 100 | -100 | -5 | | |
| 6 | -100 | -5 | -100 | -5 | | |
| 7 | -200 | -5 | 5 | 200 | | |
| 8 | 5 | 200 | 5 | 200 | | |
| 9 | 5 | 200 | -200 | -5 | | |
| 10 | -200 | -5 | -200 | -5 | | |

NODAL SYMMETRIES

NODE # WITH # ABOUT PLANE (1)XZ (2)YZ (3)XY

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 22 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
| 3 | 2 |
| 4 | 2 |
| 5 | 2 |
| 7 | 6 |
| 8 | 6 |
| 9 | 6 |
| 11 | 10 |
| 13 | 12 |
| 15 | 14 |
| 16 | 14 |
| 17 | 14 |
| 19 | 18 |
| 20 | 18 |
| 21 | 18 |
| 23 | 22 |
| 24 | 22 |
| 25 | 22 |

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

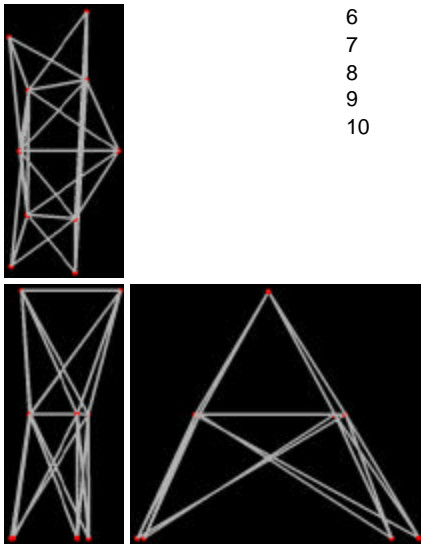
| BEAM # | SET # |
|--------|-------|
|--------|-------|

BEAM ELEMENT EQUALITIES

| BEAM # | WITH # |
|--------|--------|
|--------|--------|

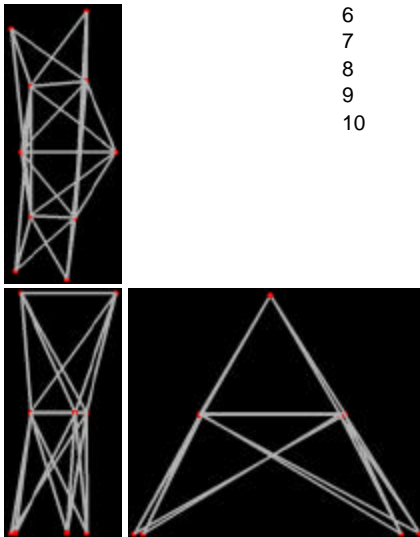
Intermediate result for study 4:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|--|----------|----------|----------|---------|-------|------|-------|----------|---|-------|---|-----|
| 1 | 0.01 | <table border="1"> <tr><td>FITNESS:</td><td>73.9687</td></tr> <tr><td>MASS:</td><td>73.9687</td></tr> <tr><td>GENS:</td><td>7046</td></tr> <tr><td>TIME:</td><td>00:23:27</td></tr> </table> | FITNESS: | 73.9687 | MASS: | 73.9687 | GENS: | 7046 | TIME: | 00:23:27 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 73.9687 | | | | | | | | | | | | | |
| MASS: | 73.9687 | | | | | | | | | | | | | |
| GENS: | 7046 | | | | | | | | | | | | | |
| TIME: | 00:23:27 | | | | | | | | | | | | | |
| 2 | 0.132632 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.132632 | 3 | -30.9317 | 49.7928 | 100 | | | | | | | | | |
| 4 | 0.132632 | 4 | 13.3563 | 58.0395 | 100 | | | | | | | | | |
| 5 | 0.132632 | | 5 | 5 | -55.1662 | 100 | | | | | | | | |
| 6 | 0.603934 | | 6 | -31.488 | -51.8469 | 100 | | | | | | | | |
| 7 | 0.603934 | | 7 | -45.3142 | 92.8815 | 0 | | | | | | | | |
| 8 | 0.603934 | | 8 | 13.7141 | 113.415 | 0 | | | | | | | | |
| 9 | 0.603934 | | 9 | 5 | -98.8093 | 0 | | | | | | | | |
| 10 | 0.01 | | 10 | -43.9303 | -93.6965 | 0 | | | | | | | | |
| 11 | 0.01 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.01 | | | | | | | | | | | | | |
| 14 | 0.0567814 | | | | | | | | | | | | | |
| 15 | 0.0567814 | | | | | | | | | | | | | |
| 16 | 0.0567814 | | | | | | | | | | | | | |
| 17 | 0.0567814 | | | | | | | | | | | | | |
| 18 | 0.23293 | | | | | | | | | | | | | |
| 19 | 0.23293 | | | | | | | | | | | | | |
| 20 | 0.23293 | | | | | | | | | | | | | |
| 21 | 0.23293 | | | | | | | | | | | | | |
| 22 | 0.532296 | | | | | | | | | | | | | |
| 23 | 0.532296 | | | | | | | | | | | | | |
| 24 | 0.532296 | | | | | | | | | | | | | |
| 25 | 0.532296 | | | | | | | | | | | | | |



Final result for study 4:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|--|----------|----------|----------|---------|-------|------|-------|----------|---|-------|---|-----|
| 1 | 0.01 | <table border="1"> <tr><td>FITNESS:</td><td>67.6458</td></tr> <tr><td>MASS:</td><td>67.6458</td></tr> <tr><td>GENS:</td><td>6296</td></tr> <tr><td>TIME:</td><td>00:21:23</td></tr> </table> | FITNESS: | 67.6458 | MASS: | 67.6458 | GENS: | 6296 | TIME: | 00:21:23 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 67.6458 | | | | | | | | | | | | | |
| MASS: | 67.6458 | | | | | | | | | | | | | |
| GENS: | 6296 | | | | | | | | | | | | | |
| TIME: | 00:21:23 | | | | | | | | | | | | | |
| 2 | 0.10816 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.10816 | 3 | -29.9035 | 54.4134 | 100 | | | | | | | | | |
| 4 | 0.10816 | 4 | 13.7177 | 58.8009 | 100 | | | | | | | | | |
| 5 | 0.10816 | | 5 | 5.05095 | -55.2474 | 100 | | | | | | | | |
| 6 | 0.595653 | | 6 | -29.71 | -53.4227 | 100 | | | | | | | | |
| 7 | 0.595653 | | 7 | -45.3142 | 102.755 | 0 | | | | | | | | |
| 8 | 0.595653 | | 8 | 14 | 117.395 | 0 | | | | | | | | |
| 9 | 0.595653 | | 9 | -1.19976 | -106.984 | 0 | | | | | | | | |
| 10 | 0.01 | | 10 | -41.9164 | -99.8283 | 0 | | | | | | | | |
| 11 | 0.01 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.01 | | | | | | | | | | | | | |
| 14 | 0.0215511 | | | | | | | | | | | | | |
| 15 | 0.0215511 | | | | | | | | | | | | | |
| 16 | 0.0215511 | | | | | | | | | | | | | |
| 17 | 0.0215511 | | | | | | | | | | | | | |
| 18 | 0.178786 | | | | | | | | | | | | | |
| 19 | 0.178786 | | | | | | | | | | | | | |
| 20 | 0.178786 | | | | | | | | | | | | | |
| 21 | 0.178786 | | | | | | | | | | | | | |
| 22 | 0.52439 | | | | | | | | | | | | | |
| 23 | 0.52439 | | | | | | | | | | | | | |
| 24 | 0.52439 | | | | | | | | | | | | | |
| 25 | 0.52439 | | | | | | | | | | | | | |



Optimization file for initial structure of study 5:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
| 4 | 5 | 100 | 5 | 100 | | |
| 8 | 5 | 200 | 5 | 200 | | |

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE (1)XZ (2)YZ (3)XY |
|--------|--------|-------------------------------|
| 3 | 4 | 2 |
| 5 | 4 | 1 |
| 6 | 3 | 1 |
| 7 | 8 | 2 |
| 9 | 8 | 1 |
| 10 | 7 | 1 |

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 3 | 0.01 | 3 |
| 4 | 0.01 | 3 |
| 5 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 7 | 0.01 | 3 |
| 8 | 0.01 | 3 |
| 9 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 11 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 13 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 15 | 0.01 | 3 |
| 16 | 0.01 | 3 |
| 17 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 19 | 0.01 | 3 |
| 20 | 0.01 | 3 |
| 21 | 0.01 | 3 |
| 22 | 0.01 | 3 |
| 23 | 0.01 | 3 |
| 24 | 0.01 | 3 |
| 25 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

TRUSS # WITH #

ALL POSSIBLE CROSS SECTIONS

SEC # TYPE GEOMETRY

CROSS SECTION SETS

SET # SECTION #

BEAM ELEMENT BOUNDARIES

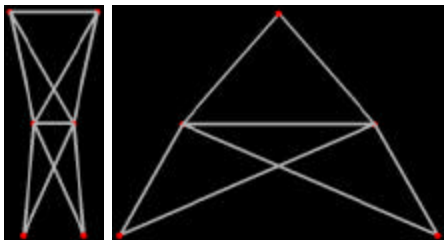
BEAM # SET #

BEAM ELEMENT EQUALITIES

BEAM # WITH #

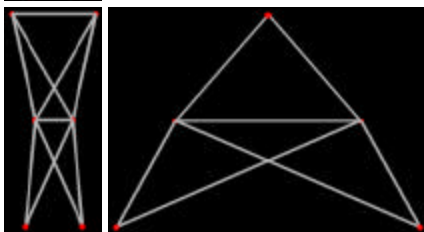
Intermediate result for study 5:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|---|----------|----------|-------|---------|-------|------|-------|-----|---|-------|---|-----|
| 1 | 0.0829865 | <table border="1"> <tr><td>FITNESS:</td><td>66.0217</td></tr> <tr><td>MASS:</td><td>66.0217</td></tr> <tr><td>GENS:</td><td>3625</td></tr> <tr><td>TIME:</td><td>504</td></tr> </table> | FITNESS: | 66.0217 | MASS: | 66.0217 | GENS: | 3625 | TIME: | 504 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 66.0217 | | | | | | | | | | | | | |
| MASS: | 66.0217 | | | | | | | | | | | | | |
| GENS: | 3625 | | | | | | | | | | | | | |
| TIME: | 504 | | | | | | | | | | | | | |
| 2 | 0.262558 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.01 | 3 | -16.7688 | 81.252 | 100 | | | | | | | | | |
| 4 | 0.285839 | 4 | 16.7688 | 81.252 | 100 | | | | | | | | | |
| 5 | 0.115668 | 5 | 16.7688 | -81.252 | 100 | | | | | | | | | |
| 6 | 0.544888 | 6 | -16.7688 | -81.252 | 100 | | | | | | | | | |
| 7 | 0.302736 | 7 | -26.2756 | 134.517 | 0 | | | | | | | | | |
| 8 | 0.334221 | 8 | 26.2756 | 134.517 | 0 | | | | | | | | | |
| 9 | 0.108235 | 9 | 26.2756 | -134.517 | 0 | | | | | | | | | |
| 10 | 0.124455 | 10 | -26.2756 | -134.517 | 0 | | | | | | | | | |
| 11 | 0.01 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.0885735 | | | | | | | | | | | | | |
| 14 | 0.01 | | | | | | | | | | | | | |
| 15 | 0.141535 | | | | | | | | | | | | | |
| 16 | 0.145808 | | | | | | | | | | | | | |
| 17 | 0.121555 | | | | | | | | | | | | | |
| 18 | 0.129828 | | | | | | | | | | | | | |
| 19 | 0.158682 | | | | | | | | | | | | | |
| 20 | 0.154423 | | | | | | | | | | | | | |
| 21 | 0.320456 | | | | | | | | | | | | | |
| 22 | 0.01 | | | | | | | | | | | | | |
| 23 | 0.173577 | | | | | | | | | | | | | |
| 24 | 0.773612 | | | | | | | | | | | | | |
| 25 | 0.573243 | | | | | | | | | | | | | |



Final result for study 5:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|---|----------|----------|-------|---------|-------|------|-------|-----|---|-------|---|-----|
| 1 | 0.0243038 | <table border="1"> <tr><td>FITNESS:</td><td>58.5302</td></tr> <tr><td>MASS:</td><td>58.5302</td></tr> <tr><td>GENS:</td><td>4408</td></tr> <tr><td>TIME:</td><td>543</td></tr> </table> | FITNESS: | 58.5302 | MASS: | 58.5302 | GENS: | 4408 | TIME: | 543 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 58.5302 | | | | | | | | | | | | | |
| MASS: | 58.5302 | | | | | | | | | | | | | |
| GENS: | 4408 | | | | | | | | | | | | | |
| TIME: | 543 | | | | | | | | | | | | | |
| 2 | 0.248379 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.0258587 | 3 | -16.5988 | 80.8489 | 100 | | | | | | | | | |
| 4 | 0.282607 | 4 | 16.5988 | 80.8489 | 100 | | | | | | | | | |
| 5 | 0.103001 | 5 | 16.5988 | -80.8489 | 100 | | | | | | | | | |
| 6 | 0.528236 | 6 | -16.5988 | -80.8489 | 100 | | | | | | | | | |
| 7 | 0.29108 | 7 | -25.7349 | 132.74 | 0 | | | | | | | | | |
| 8 | 0.340559 | 8 | 25.7349 | 132.74 | 0 | | | | | | | | | |
| 9 | 0.103114 | 9 | 25.7349 | -132.74 | 0 | | | | | | | | | |
| 10 | 0.0197579 | 10 | -25.7349 | -132.74 | 0 | | | | | | | | | |
| 11 | 0.01 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.133679 | | | | | | | | | | | | | |
| 14 | 0.0696404 | | | | | | | | | | | | | |
| 15 | 0.0398215 | | | | | | | | | | | | | |
| 16 | 0.144947 | | | | | | | | | | | | | |
| 17 | 0.115036 | | | | | | | | | | | | | |
| 18 | 0.122048 | | | | | | | | | | | | | |
| 19 | 0.149425 | | | | | | | | | | | | | |
| 20 | 0.147719 | | | | | | | | | | | | | |
| 21 | 0.22 | | | | | | | | | | | | | |
| 22 | 0.01 | | | | | | | | | | | | | |
| 23 | 0.208012 | | | | | | | | | | | | | |
| 24 | 0.667593 | | | | | | | | | | | | | |
| 25 | 0.469535 | | | | | | | | | | | | | |



Optimization file for initial structure of study 6:

NODAL BOUNDARIES

| NODE # | X-LO | X-HI | Y-LO | Y-HI | Z-LO | Z-HI |
|--------|------|------|------|------|------|------|
| 3 | -100 | -5 | 5 | 100 | | |
| 4 | 5 | 100 | 5 | 100 | | |
| 5 | 5 | 100 | -100 | -5 | | |
| 6 | -100 | -5 | -100 | -5 | | |
| 7 | -200 | -5 | 5 | 200 | | |
| 8 | 5 | 200 | 5 | 200 | | |
| 9 | 5 | 200 | -200 | -5 | | |
| 10 | -200 | -5 | -200 | -5 | | |

NODAL SYMMETRIES

| NODE # | WITH # | ABOUT PLANE (1)XZ (2)YZ (3)XY |
|--------|--------|-------------------------------|
|--------|--------|-------------------------------|

DISPLACEMENT BOUNDARIES

| NODE # | DX | DY | DZ | RX | RY | RZ |
|--------|----|----|----|----|----|----|
| 1 | 2 | 2 | 2 | | | |
| 2 | 2 | 2 | 2 | | | |
| 3 | 2 | 2 | 2 | | | |
| 4 | 2 | 2 | 2 | | | |
| 5 | 2 | 2 | 2 | | | |
| 6 | 2 | 2 | 2 | | | |

TRUSS ELEMENT BOUNDARIES

| TRUSS # | A-LO | A-HI |
|---------|------|------|
| 1 | 0.01 | 3 |
| 2 | 0.01 | 3 |
| 3 | 0.01 | 3 |
| 4 | 0.01 | 3 |
| 5 | 0.01 | 3 |
| 6 | 0.01 | 3 |
| 7 | 0.01 | 3 |
| 8 | 0.01 | 3 |
| 9 | 0.01 | 3 |
| 10 | 0.01 | 3 |
| 11 | 0.01 | 3 |
| 12 | 0.01 | 3 |
| 13 | 0.01 | 3 |
| 14 | 0.01 | 3 |
| 15 | 0.01 | 3 |
| 16 | 0.01 | 3 |
| 17 | 0.01 | 3 |
| 18 | 0.01 | 3 |
| 19 | 0.01 | 3 |
| 20 | 0.01 | 3 |
| 21 | 0.01 | 3 |
| 22 | 0.01 | 3 |
| 23 | 0.01 | 3 |
| 24 | 0.01 | 3 |
| 25 | 0.01 | 3 |

TRUSS ELEMENT EQUALITIES

| TRUSS # | WITH # |
|---------|--------|
|---------|--------|

ALL POSSIBLE CROSS SECTIONS

| SEC # | TYPE | GEOMETRY |
|-------|------|----------|
|-------|------|----------|

CROSS SECTION SETS

| SET # | SECTION # |
|-------|-----------|
|-------|-----------|

BEAM ELEMENT BOUNDARIES

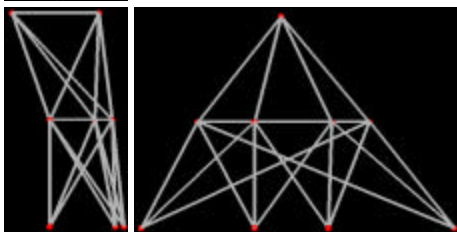
| BEAM # | SET # |
|--------|-------|
|--------|-------|

BEAM ELEMENT EQUALITIES

| BEAM # | WITH # |
|--------|--------|
|--------|--------|

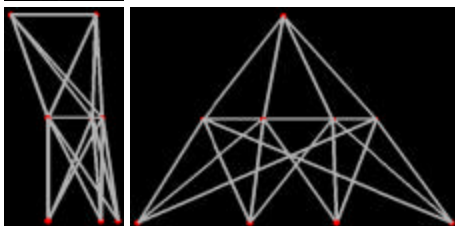
Intermediate result for study 6:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|--|----------|----------|-------|---------|-------|------|-------|----------|---|-------|---|-----|
| 1 | 0.0485342 | <table border="1"> <tr><td>FITNESS:</td><td>60.0886</td></tr> <tr><td>MASS:</td><td>60.0886</td></tr> <tr><td>GENS:</td><td>6170</td></tr> <tr><td>TIME:</td><td>00:20:09</td></tr> </table> | FITNESS: | 60.0886 | MASS: | 60.0886 | GENS: | 6170 | TIME: | 00:20:09 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 60.0886 | | | | | | | | | | | | | |
| MASS: | 60.0886 | | | | | | | | | | | | | |
| GENS: | 6170 | | | | | | | | | | | | | |
| TIME: | 00:20:09 | | | | | | | | | | | | | |
| 2 | 0.01 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.54628 | 3 | -5 | 77.488 | 100 | | | | | | | | | |
| 4 | 0.01 | 4 | 34.0829 | 44.4317 | 100 | | | | | | | | | |
| 5 | 0.412293 | 5 | 49.3698 | -23.0936 | 100 | | | | | | | | | |
| 6 | 0.165335 | 6 | -5 | -72.6242 | 100 | | | | | | | | | |
| 7 | 0.01 | 7 | -5 | 151.56 | 0 | | | | | | | | | |
| 8 | 0.583618 | 8 | 59.2451 | 41.3649 | 0 | | | | | | | | | |
| 9 | 0.396922 | 9 | 51.737 | -23.127 | 0 | | | | | | | | | |
| 10 | 0.153436 | 10 | -5 | -122.172 | 0 | | | | | | | | | |
| 11 | 0.01 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.01 | | | | | | | | | | | | | |
| 14 | 0.0758236 | | | | | | | | | | | | | |
| 15 | 0.01 | | | | | | | | | | | | | |
| 16 | 0.01 | | | | | | | | | | | | | |
| 17 | 0.01 | | | | | | | | | | | | | |
| 18 | 0.0895946 | | | | | | | | | | | | | |
| 19 | 0.0640194 | | | | | | | | | | | | | |
| 20 | 0.01 | | | | | | | | | | | | | |
| 21 | 0.121732 | | | | | | | | | | | | | |
| 22 | 0.739591 | | | | | | | | | | | | | |
| 23 | 1.15299 | | | | | | | | | | | | | |
| 24 | 0.09338 | | | | | | | | | | | | | |
| 25 | 0.01 | | | | | | | | | | | | | |



Final result for study 6:

| ELEMENT | AREA | RESULTS | NODE | X-POS | Y-POS | Z-POS | | | | | | | | |
|----------|-----------|--|----------|----------|-------|---------|-------|------|-------|----------|---|-------|---|-----|
| 1 | 0.0473182 | <table border="1"> <tr><td>FITNESS:</td><td>56.1709</td></tr> <tr><td>MASS:</td><td>56.1709</td></tr> <tr><td>GENS:</td><td>3923</td></tr> <tr><td>TIME:</td><td>00:12:44</td></tr> </table> | FITNESS: | 56.1709 | MASS: | 56.1709 | GENS: | 3923 | TIME: | 00:12:44 | 1 | -37.5 | 0 | 200 |
| FITNESS: | 56.1709 | | | | | | | | | | | | | |
| MASS: | 56.1709 | | | | | | | | | | | | | |
| GENS: | 3923 | | | | | | | | | | | | | |
| TIME: | 00:12:44 | | | | | | | | | | | | | |
| 2 | 0.01 | 2 | 37.5 | 0 | 200 | | | | | | | | | |
| 3 | 0.544446 | 3 | -4.8178 | 83.501 | 100 | | | | | | | | | |
| 4 | 0.01 | 4 | 34.1034 | 44.1595 | 100 | | | | | | | | | |
| 5 | 0.39114 | 5 | 42.9971 | -18.0179 | 100 | | | | | | | | | |
| 6 | 0.120537 | 6 | -5.0001 | -72.6017 | 100 | | | | | | | | | |
| 7 | 0.01 | 7 | -4.53035 | 151.93 | 0 | | | | | | | | | |
| 8 | 0.575429 | 8 | 57.1199 | 47.5462 | 0 | | | | | | | | | |
| 9 | 0.366997 | 9 | 42 | -30.0061 | 0 | | | | | | | | | |
| 10 | 0.0646268 | 10 | -5 | -131.882 | 0 | | | | | | | | | |
| 11 | 0.0115837 | | | | | | | | | | | | | |
| 12 | 0.01 | | | | | | | | | | | | | |
| 13 | 0.0123771 | | | | | | | | | | | | | |
| 14 | 0.066825 | | | | | | | | | | | | | |
| 15 | 0.01 | | | | | | | | | | | | | |
| 16 | 0.01 | | | | | | | | | | | | | |
| 17 | 0.01 | | | | | | | | | | | | | |
| 18 | 0.0432764 | | | | | | | | | | | | | |
| 19 | 0.0544915 | | | | | | | | | | | | | |
| 20 | 0.01 | | | | | | | | | | | | | |
| 21 | 0.0838608 | | | | | | | | | | | | | |
| 22 | 0.784988 | | | | | | | | | | | | | |
| 23 | 1.08439 | | | | | | | | | | | | | |
| 24 | 0.0793082 | | | | | | | | | | | | | |
| 25 | 0.0281745 | | | | | | | | | | | | | |



10.0 APPENDIX C: MODEL DATA FOR THE FSAE CAR FRAME

Input files for the initial structure:

| NUMBER | X-POS | Y-POS | Z-POS | X-CONS | Y-CONS | Z-CONS | Rx-CONS | Ry-CONS | Rz-CONS |
|--------|----------|---------|---------|--------|--------|--------|---------|---------|---------|
| 1 | -7 | 4.5 | 93 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 7 | 4.5 | 93 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | -6 | 16.5 | 93 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 6 | 16.5 | 93 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | -7 | 4.5 | 78 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 7 | 4.5 | 78 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | -10 | 12.5 | 77.1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 10 | 12.5 | 77.1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | -7 | 4.5 | 76.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 | 7 | 4.5 | 76.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | -10 | 12.5 | 75.6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 10 | 12.5 | 75.6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | -7 | 4.5 | 74.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 7 | 4.5 | 74.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | -7 | 4.5 | 65.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 7 | 4.5 | 65.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | -10 | 12.5 | 66.25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 10 | 12.5 | 66.25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | -7 | 4.5 | 64 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 7 | 4.5 | 64 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | -8.3068 | 7.9847 | 64.3267 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 8.3068 | 7.9847 | 64.3267 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | -6 | 7.9847 | 64.3267 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 6 | 7.9847 | 64.3267 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | -10 | 12.5 | 64.75 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 10 | 12.5 | 64.75 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | -4.5 | 22.4016 | 65.6783 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 4.5 | 22.4016 | 65.6783 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 4.5 | 74.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 22.4016 | 65.6783 | 0 | 0 | 0 | 0 | 0 | 0 |
| 31 | -8.1786 | 4.1071 | 59.0155 | 0 | 0 | 0 | 0 | 0 | 0 |
| 32 | 8.1786 | 4.1071 | 59.0155 | 0 | 0 | 0 | 0 | 0 | 0 |
| 33 | -6 | 4.1071 | 59.0155 | 0 | 0 | 0 | 0 | 0 | 0 |
| 34 | 6 | 4.1071 | 59.0155 | 0 | 0 | 0 | 0 | 0 | 0 |
| 35 | -10.4762 | 3.3413 | 49.2985 | 0 | 0 | 0 | 0 | 0 | 0 |
| 36 | 10.4762 | 3.3413 | 49.2985 | 0 | 0 | 0 | 0 | 0 | 0 |
| 37 | -6 | 3.3413 | 49.2985 | 0 | 0 | 0 | 0 | 0 | 0 |
| 38 | 6 | 3.3413 | 49.2985 | 0 | 0 | 0 | 0 | 0 | 0 |
| 39 | -13 | 2.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 40 | 13 | 2.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 41 | -13 | 12.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 | 13 | 12.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 43 | -13 | 20.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 | 13 | 20.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 45 | -6 | 16.3912 | 33.5019 | 0 | 0 | 0 | 0 | 0 | 0 |
| 46 | 6 | 16.3912 | 33.5019 | 0 | 0 | 0 | 0 | 0 | 0 |
| 47 | -2.925 | 16.3912 | 33.5019 | 0 | 0 | 0 | 0 | 0 | 0 |
| 48 | 2.925 | 16.3912 | 33.5019 | 0 | 0 | 0 | 0 | 0 | 0 |
| 49 | -8.062 | 16.1798 | 30.4784 | 0 | 0 | 0 | 0 | 0 | 0 |
| 50 | 8.062 | 16.1798 | 30.4784 | 0 | 0 | 0 | 0 | 0 | 0 |
| 51 | -6.3824 | 33 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 52 | 6.3824 | 33 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 53 | -4 | 37.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 54 | 4 | 37.5 | 38.625 | 0 | 0 | 0 | 0 | 0 | 0 |
| 55 | -7 | 4.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 56 | 7 | 4.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|-----|---------|--------|--------|---|---|---|---|---|---|
| 57 | -2.61 | 4.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 58 | 3.022 | 4.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 59 | -10 | 12.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 60 | 10 | 12.5 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 61 | -4.5 | 16.235 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 62 | 4.5 | 16.235 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 63 | -3.282 | 16.235 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 64 | 2.338 | 16.235 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 65 | -7 | 4.5 | 12.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 66 | 7 | 4.5 | 12.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 67 | -10 | 12.5 | 12.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 68 | 10 | 12.5 | 12.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 69 | -7 | 4.5 | 3.814 | 0 | 0 | 0 | 0 | 0 | 0 |
| 70 | 7 | 4.5 | 3.814 | 0 | 0 | 0 | 0 | 0 | 0 |
| 71 | -7 | 4.5 | 2.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 72 | 7 | 4.5 | 2.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 73 | -10 | 12.5 | 2.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 74 | 10 | 12.5 | 2.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 75 | -7 | 4.5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 76 | 7 | 4.5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 77 | -10 | 12.5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 78 | 10 | 12.5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 79 | 0 | 4.5 | 3.814 | 0 | 0 | 0 | 0 | 0 | 0 |
| 80 | -2.61 | 5.75 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 81 | 3.022 | 5.75 | 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 82 | -3.282 | 15 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 83 | 2.338 | 15 | 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 84 | -8.5 | 4.63 | 74.7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 85 | 8.5 | 4.63 | 74.7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 86 | -8.5 | 4.63 | 73.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 87 | 8.5 | 4.63 | 73.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 88 | -8.5 | 4.6 | 5.3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 89 | 8.5 | 4.6 | 5.3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 90 | -8.5 | 4.6 | 4.4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 91 | 8.5 | 4.6 | 4.4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 92 | -9.685 | 3.337 | 74.174 | 0 | 0 | 0 | 1 | 1 | 1 |
| 93 | 9.685 | 3.337 | 74.174 | 0 | 0 | 0 | 1 | 1 | 1 |
| 94 | -10 | 8.474 | 74.2 | 0 | 0 | 0 | 1 | 1 | 1 |
| 95 | 10 | 8.474 | 74.2 | 0 | 0 | 0 | 1 | 1 | 1 |
| 96 | -9.634 | 3.2 | 5.632 | 0 | 0 | 0 | 1 | 1 | 1 |
| 97 | 9.634 | 3.2 | 5.632 | 0 | 0 | 0 | 1 | 1 | 1 |
| 98 | -10.268 | 8.215 | 5.782 | 0 | 0 | 0 | 1 | 1 | 1 |
| 99 | 10.268 | 8.215 | 5.782 | 0 | 0 | 0 | 1 | 1 | 1 |
| 100 | -24 | 4.35 | 74.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 101 | 24 | 4.35 | 74.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 102 | -23.636 | 5.776 | 70.381 | 0 | 0 | 0 | 0 | 0 | 0 |
| 103 | 23.636 | 5.776 | 70.381 | 0 | 0 | 0 | 0 | 0 | 0 |
| 104 | -24.033 | 14.808 | 73.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 105 | 24.033 | 14.808 | 73.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 106 | -23 | 4 | 8.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 107 | 23 | 4 | 8.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 108 | -23 | 5.154 | 5.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 109 | 23 | 5.154 | 5.5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 110 | -23 | 14.5 | 8.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 111 | 23 | 14.5 | 8.8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 112 | -23.5 | 10 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 113 | 23.5 | 10 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 114 | -24 | 9.75 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 115 | 24 | 9.75 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 116 | -26.5 | 10 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 117 | 26.5 | 10 | 74 | 0 | 0 | 0 | 0 | 0 | 0 |
| 118 | -25 | 0 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| 119 | 25 | 0 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| 120 | -22.5 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|-----|-------|------|--------|---|---|---|---|---|---|
| 121 | 22.5 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 122 | -24 | 9.75 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 123 | 24 | 9.75 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 124 | -25.5 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 125 | 25.5 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 126 | -24 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 127 | 24 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 128 | 4.3 | 11.3 | 15.9 | 0 | 0 | 0 | 0 | 0 | 0 |
| 129 | -7 | -20 | 38.625 | 1 | 1 | 1 | 1 | 1 | 1 |
| 130 | 7 | -20 | 38.625 | 1 | 1 | 1 | 1 | 1 | 1 |
| 131 | 0 | -20 | 54 | 1 | 1 | 1 | 1 | 1 | 1 |
| 132 | 0 | -20 | 24 | 1 | 1 | 1 | 1 | 1 | 1 |

| NUMBER | TYPE | NODE1 | NODE2 | NODE_K | MATL | PROPERTIES |
|--------|-----------|-------|-------|--------|------|-------------------|
| | (1) TRUSS | | | | | AREA |
| | (2) BEAM | | | | | SEC TYPE GEOMETRY |
| 1 | 1 | 84 | 92 | | | 2 1 |
| 2 | 1 | 85 | 93 | | | 2 1 |
| 3 | 1 | 84 | 94 | | | 2 1 |
| 4 | 1 | 85 | 95 | | | 2 1 |
| 5 | 1 | 86 | 92 | | | 2 1 |
| 6 | 1 | 87 | 93 | | | 2 1 |
| 7 | 1 | 86 | 94 | | | 2 1 |
| 8 | 1 | 87 | 95 | | | 2 1 |
| 9 | 1 | 92 | 94 | | | 2 1 |
| 10 | 1 | 93 | 95 | | | 2 1 |
| 11 | 1 | 88 | 96 | | | 2 1 |
| 12 | 1 | 89 | 97 | | | 2 1 |
| 13 | 1 | 88 | 98 | | | 2 1 |
| 14 | 1 | 89 | 99 | | | 2 1 |
| 15 | 1 | 90 | 96 | | | 2 1 |
| 16 | 1 | 91 | 97 | | | 2 1 |
| 17 | 1 | 90 | 98 | | | 2 1 |
| 18 | 1 | 91 | 99 | | | 2 1 |
| 19 | 1 | 96 | 98 | | | 2 1 |
| 20 | 1 | 97 | 99 | | | 2 1 |
| 21 | 1 | 9 | 100 | | | 2 0.2 |
| 22 | 1 | 10 | 101 | | | 2 0.2 |
| 23 | 1 | 15 | 100 | | | 2 0.2 |
| 24 | 1 | 16 | 101 | | | 2 0.2 |
| 25 | 1 | 29 | 102 | | | 2 0.2 |
| 26 | 1 | 29 | 103 | | | 2 0.2 |
| 27 | 1 | 11 | 104 | | | 2 0.2 |
| 28 | 1 | 12 | 105 | | | 2 0.2 |
| 29 | 1 | 17 | 104 | | | 2 0.2 |
| 30 | 1 | 18 | 105 | | | 2 0.2 |
| 31 | 1 | 65 | 106 | | | 2 0.2 |
| 32 | 1 | 66 | 107 | | | 2 0.2 |
| 33 | 1 | 71 | 106 | | | 2 0.2 |
| 34 | 1 | 72 | 107 | | | 2 0.2 |
| 35 | 1 | 71 | 108 | | | 2 0.2 |
| 36 | 1 | 72 | 109 | | | 2 0.2 |
| 37 | 1 | 67 | 110 | | | 2 0.2 |
| 38 | 1 | 68 | 111 | | | 2 0.2 |
| 39 | 1 | 73 | 110 | | | 2 0.2 |
| 40 | 1 | 74 | 111 | | | 2 0.2 |
| 41 | 1 | 29 | 94 | | | 2 0.2 |
| 42 | 1 | 29 | 95 | | | 2 0.2 |
| 43 | 1 | 92 | 104 | | | 2 0.2 |
| 44 | 1 | 93 | 105 | | | 2 0.2 |
| 45 | 1 | 79 | 98 | | | 2 0.2 |
| 46 | 1 | 79 | 99 | | | 2 0.2 |
| 47 | 1 | 96 | 110 | | | 2 0.2 |

| | | | | | | | | | |
|----|---|-----|------|---|-----|------|--|-------|-------|
| 48 | 1 | 97 | 111 | 2 | 0.2 | | | | |
| 49 | 1 | 84 | 86 | 2 | 1 | | | | |
| 50 | 1 | 85 | 87 | 2 | 1 | | | | |
| 51 | 1 | 88 | 90 | 2 | 1 | | | | |
| 52 | 1 | 89 | 91 | 2 | 1 | | | | |
| 53 | 1 | 129 | 131 | 3 | 0.1 | | | | |
| 54 | 1 | 130 | 131 | 3 | 0.1 | | | | |
| 55 | 1 | 129 | 132 | 3 | 0.1 | | | | |
| 56 | 1 | 130 | 132 | 3 | 0.1 | | | | |
| 57 | 1 | 19 | 129 | 3 | 0.1 | | | | |
| 58 | 1 | 39 | 129 | 3 | 0.1 | | | | |
| 59 | 1 | 40 | 129 | 3 | 0.1 | | | | |
| 60 | 1 | 55 | 129 | 3 | 0.1 | | | | |
| 61 | 1 | 20 | 130 | 3 | 0.1 | | | | |
| 62 | 1 | 39 | 130 | 3 | 0.1 | | | | |
| 63 | 1 | 40 | 130 | 3 | 0.1 | | | | |
| 64 | 1 | 56 | 130 | 3 | 0.1 | | | | |
| 65 | 1 | 5 | 131 | 3 | 0.1 | | | | |
| 66 | 1 | 6 | 131 | 3 | 0.1 | | | | |
| 67 | 1 | 19 | 131 | 3 | 0.1 | | | | |
| 68 | 1 | 20 | 131 | 3 | 0.1 | | | | |
| 69 | 1 | 55 | 132 | 3 | 0.1 | | | | |
| 70 | 1 | 56 | 132 | 3 | 0.1 | | | | |
| 71 | 1 | 75 | 132 | 3 | 0.1 | | | | |
| 72 | 1 | 76 | 132 | 3 | 0.1 | | | | |
| 1 | 2 | 1 | 2 y | 1 | 3 | 1 | | 0.065 | |
| 2 | 2 | 3 | 4 y | 1 | 3 | 1 | | 0.065 | |
| 3 | 2 | 1 | 3 y | 1 | 3 | 1 | | 0.065 | |
| 4 | 2 | 2 | 4 y | 1 | 3 | 1 | | 0.065 | |
| 5 | 2 | 1 | 5 y | 1 | 3 | 1 | | 0.035 | |
| 6 | 2 | 2 | 6 y | 1 | 3 | 1 | | 0.035 | |
| 7 | 2 | 1 | 7 y | 1 | 3 | 0.5 | | 0.035 | |
| 8 | 2 | 2 | 8 y | 1 | 3 | 0.5 | | 0.035 | |
| 9 | 2 | 3 | 7 y | 1 | 3 | 0.5 | | 0.035 | |
| 10 | 2 | 4 | 8 y | 1 | 3 | 0.5 | | 0.035 | |
| 11 | 2 | 3 | 27 v | 1 | 3 | 1 | | 0.065 | |
| 12 | 2 | 4 | 28 y | 1 | 3 | 1 | | 0.065 | |
| 13 | 2 | 5 | 7 y | 1 | 3 | 1 | | 0.058 | |
| 14 | 2 | 6 | 8 y | 1 | 3 | 1 | | 0.058 | |
| 15 | 2 | 7 | 27 v | 1 | 3 | 1 | | 0.035 | |
| 16 | 2 | 8 | 28 y | 1 | 3 | 1 | | 0.035 | |
| 17 | 2 | 5 | 9 y | 1 | 4 | 1 | | 1 | 0.065 |
| 18 | 2 | 6 | 10 y | 1 | 4 | 1 | | 1 | 0.065 |
| 19 | 2 | 9 | 13 v | 1 | 4 | 1 | | 1 | 0.065 |
| 20 | 2 | 10 | 14 y | 1 | 4 | 1 | | 1 | 0.065 |
| 21 | 2 | 5 | 29 y | 1 | 3 | 0.5 | | 0.035 | |
| 22 | 2 | 6 | 29 v | 1 | 3 | 0.5 | | 0.035 | |
| 23 | 2 | 13 | 29 y | 1 | 4 | 1 | | 1 | 0.065 |
| 24 | 2 | 14 | 29 y | 1 | 4 | 1 | | 1 | 0.065 |
| 25 | 2 | 13 | 15 y | 1 | 4 | 1 | | 1 | 0.065 |
| 26 | 2 | 14 | 16 v | 1 | 4 | 1 | | 1 | 0.065 |
| 27 | 2 | 19 | 29 y | 1 | 3 | 0.5 | | 0.035 | |
| 28 | 2 | 20 | 29 y | 1 | 3 | 0.5 | | 0.035 | |
| 29 | 2 | 15 | 19 y | 1 | 4 | 1 | | 1 | 0.065 |
| 30 | 2 | 16 | 20 v | 1 | 4 | 1 | | 1 | 0.065 |
| 31 | 2 | 13 | 25 y | 1 | 3 | 0.75 | | 0.035 | |
| 32 | 2 | 14 | 26 y | 1 | 3 | 0.75 | | 0.035 | |
| 33 | 2 | 7 | 11 y | 1 | 4 | 1 | | 1 | 0.065 |
| 34 | 2 | 8 | 12 v | 1 | 4 | 1 | | 1 | 0.065 |
| 35 | 2 | 11 | 17 y | 1 | 4 | 1 | | 1 | 0.065 |
| 36 | 2 | 12 | 18 y | 1 | 4 | 1 | | 1 | 0.065 |
| 37 | 2 | 17 | 25 y | 1 | 4 | 1 | | 1 | 0.065 |
| 38 | 2 | 18 | 26 v | 1 | 4 | 1 | | 1 | 0.065 |
| 39 | 2 | 19 | 21 y | 1 | 3 | 1 | | 0.095 | |

| | | | | | | | |
|-----|---|----|------|---|---|-------|-------|
| 40 | 2 | 20 | 22 v | 1 | 3 | 1 | 0.095 |
| 41 | 2 | 21 | 25 v | 1 | 3 | 1 | 0.095 |
| 42 | 2 | 22 | 26 v | 1 | 3 | 1 | 0.095 |
| 43 | 2 | 21 | 23 v | 1 | 3 | 1 | 0.065 |
| 44 | 2 | 22 | 24 y | 1 | 3 | 1 | 0.065 |
| 45 | 2 | 25 | 27 y | 1 | 3 | 1 | 0.095 |
| 46 | 2 | 26 | 28 y | 1 | 3 | 1 | 0.095 |
| 47 | 2 | 27 | 30 y | 1 | 3 | 1 | 0.095 |
| 48 | 2 | 28 | 30 y | 1 | 3 | 1 | 0.095 |
| 49 | 2 | 29 | 30 v | 1 | 3 | 0.625 | 0.035 |
| 50 | 2 | 23 | 24 v | 1 | 3 | 1 | 0.065 |
| 51 | 2 | 5 | 6 v | 1 | 3 | 0.5 | 0.035 |
| 52 | 2 | 19 | 20 y | 1 | 3 | 0.5 | 0.035 |
| 53 | 2 | 19 | 31 y | 1 | 3 | 1 | 0.065 |
| 54 | 2 | 20 | 32 y | 1 | 3 | 1 | 0.065 |
| 55 | 2 | 31 | 33 y | 1 | 3 | 0.75 | 0.035 |
| 56 | 2 | 32 | 34 y | 1 | 3 | 0.75 | 0.035 |
| 57 | 2 | 31 | 35 v | 1 | 3 | 1 | 0.065 |
| 58 | 2 | 32 | 36 v | 1 | 3 | 1 | 0.065 |
| 59 | 2 | 35 | 37 v | 1 | 3 | 0.75 | 0.035 |
| 60 | 2 | 36 | 38 y | 1 | 3 | 0.75 | 0.035 |
| 61 | 2 | 33 | 34 y | 1 | 3 | 0.75 | 0.035 |
| 62 | 2 | 37 | 38 y | 1 | 3 | 0.75 | 0.035 |
| 63 | 2 | 35 | 39 y | 1 | 3 | 1 | 0.065 |
| 64 | 2 | 36 | 40 y | 1 | 3 | 1 | 0.065 |
| 65 | 2 | 25 | 39 y | 1 | 3 | 1 | 0.065 |
| 66 | 2 | 26 | 40 v | 1 | 3 | 1 | 0.065 |
| 67 | 2 | 25 | 41 v | 1 | 3 | 1 | 0.065 |
| 68 | 2 | 26 | 42 v | 1 | 3 | 1 | 0.065 |
| 69 | 2 | 25 | 43 y | 1 | 3 | 0.5 | 0.035 |
| 70 | 2 | 26 | 44 y | 1 | 3 | 0.5 | 0.035 |
| 71 | 2 | 27 | 43 y | 1 | 3 | 0.75 | 0.035 |
| 72 | 2 | 28 | 44 y | 1 | 3 | 0.75 | 0.035 |
| 73 | 2 | 39 | 41 y | 1 | 3 | 1 | 0.095 |
| 74 | 2 | 40 | 42 v | 1 | 3 | 1 | 0.095 |
| 75 | 2 | 41 | 43 v | 1 | 3 | 1 | 0.095 |
| 76 | 2 | 42 | 44 v | 1 | 3 | 1 | 0.095 |
| 77 | 2 | 39 | 49 y | 1 | 3 | 0.75 | 0.035 |
| 78 | 2 | 40 | 50 y | 1 | 3 | 0.75 | 0.035 |
| 79 | 2 | 41 | 49 y | 1 | 3 | 0.75 | 0.035 |
| 80 | 2 | 42 | 50 y | 1 | 3 | 0.75 | 0.035 |
| 81 | 2 | 45 | 49 y | 1 | 3 | 0.625 | 0.035 |
| 82 | 2 | 46 | 50 v | 1 | 3 | 0.625 | 0.035 |
| 83 | 2 | 43 | 45 v | 1 | 3 | 1 | 0.065 |
| 84 | 2 | 44 | 46 v | 1 | 3 | 1 | 0.065 |
| 85 | 2 | 45 | 47 v | 1 | 3 | 1 | 0.065 |
| 86 | 2 | 46 | 48 y | 1 | 3 | 1 | 0.065 |
| 87 | 2 | 39 | 40 y | 1 | 3 | 0.5 | 0.035 |
| 88 | 2 | 47 | 48 y | 1 | 3 | 1 | 0.065 |
| 89 | 2 | 43 | 51 y | 1 | 3 | 1 | 0.095 |
| 90 | 2 | 44 | 52 y | 1 | 3 | 1 | 0.095 |
| 91 | 2 | 51 | 53 v | 1 | 3 | 1 | 0.095 |
| 92 | 2 | 52 | 54 v | 1 | 3 | 1 | 0.095 |
| 93 | 2 | 51 | 52 v | 1 | 3 | 0.5 | 0.035 |
| 94 | 2 | 53 | 54 y | 1 | 3 | 1 | 0.095 |
| 95 | 2 | 39 | 55 y | 1 | 3 | 0.5 | 0.035 |
| 96 | 2 | 40 | 56 y | 1 | 3 | 0.5 | 0.035 |
| 97 | 2 | 39 | 59 y | 1 | 3 | 0.5 | 0.035 |
| 98 | 2 | 40 | 60 y | 1 | 3 | 0.5 | 0.035 |
| 99 | 2 | 43 | 59 v | 1 | 3 | 0.625 | 0.035 |
| 100 | 2 | 44 | 60 v | 1 | 3 | 0.625 | 0.035 |
| 101 | 2 | 51 | 59 v | 1 | 3 | 1 | 0.065 |
| 102 | 2 | 52 | 60 y | 1 | 3 | 1 | 0.065 |
| 103 | 2 | 51 | 60 y | 1 | 3 | 0.5 | 0.035 |
| 104 | 2 | 52 | 59 y | 1 | 3 | 0.5 | 0.035 |

| | | | | | | | | |
|-----|---|----|------|---|---|-------|-------|-------|
| 105 | 2 | 55 | 57 v | 1 | 3 | 1 | 0.065 | |
| 106 | 2 | 56 | 58 v | 1 | 3 | 1 | 0.065 | |
| 107 | 2 | 55 | 59 v | 1 | 3 | 1 | 0.035 | |
| 108 | 2 | 56 | 60 v | 1 | 3 | 1 | 0.035 | |
| 109 | 2 | 59 | 61 y | 1 | 3 | 1 | 0.065 | |
| 110 | 2 | 60 | 62 y | 1 | 3 | 1 | 0.065 | |
| 111 | 2 | 61 | 63 y | 1 | 3 | 1 | 0.065 | |
| 112 | 2 | 62 | 64 y | 1 | 3 | 1 | 0.065 | |
| 113 | 2 | 57 | 58 y | 1 | 3 | 1 | 0.065 | |
| 114 | 2 | 63 | 64 v | 1 | 3 | 1 | 0.065 | |
| 115 | 2 | 55 | 65 v | 1 | 4 | 1 | 1 | 0.065 |
| 116 | 2 | 56 | 66 v | 1 | 4 | 1 | 1 | 0.065 |
| 117 | 2 | 59 | 67 y | 1 | 4 | 1 | 1 | 0.065 |
| 118 | 2 | 60 | 68 y | 1 | 4 | 1 | 1 | 0.065 |
| 119 | 2 | 65 | 69 y | 1 | 4 | 1 | 1 | 0.065 |
| 120 | 2 | 66 | 70 y | 1 | 4 | 1 | 1 | 0.065 |
| 121 | 2 | 59 | 69 y | 1 | 3 | 0.5 | 0.035 | |
| 122 | 2 | 60 | 70 v | 1 | 3 | 0.5 | 0.035 | |
| 123 | 2 | 67 | 73 v | 1 | 4 | 1 | 1 | 0.065 |
| 124 | 2 | 68 | 74 v | 1 | 4 | 1 | 1 | 0.065 |
| 125 | 2 | 69 | 79 y | 1 | 4 | 1 | 1 | 0.065 |
| 126 | 2 | 70 | 79 y | 1 | 4 | 1 | 1 | 0.065 |
| 127 | 2 | 69 | 71 y | 1 | 4 | 1 | 1 | 0.065 |
| 128 | 2 | 70 | 72 y | 1 | 4 | 1 | 1 | 0.065 |
| 129 | 2 | 71 | 75 y | 1 | 4 | 1 | 1 | 0.065 |
| 130 | 2 | 72 | 76 y | 1 | 4 | 1 | 1 | 0.065 |
| 131 | 2 | 69 | 77 v | 1 | 3 | 0.5 | 0.035 | |
| 132 | 2 | 70 | 78 v | 1 | 3 | 0.5 | 0.035 | |
| 133 | 2 | 77 | 79 v | 1 | 3 | 0.5 | 0.035 | |
| 134 | 2 | 78 | 79 y | 1 | 3 | 0.5 | 0.035 | |
| 135 | 2 | 73 | 77 y | 1 | 4 | 1 | 1 | 0.065 |
| 136 | 2 | 74 | 78 y | 1 | 4 | 1 | 1 | 0.065 |
| 137 | 2 | 75 | 77 y | 1 | 3 | 0.5 | 0.035 | |
| 138 | 2 | 76 | 78 y | 1 | 3 | 0.5 | 0.035 | |
| 139 | 2 | 75 | 76 v | 1 | 3 | 1 | 0.035 | |
| 140 | 2 | 77 | 78 v | 1 | 3 | 0.625 | 0.035 | |
| 141 | 2 | 57 | 80 z | 2 | 2 | 0.125 | 1 | |
| 142 | 2 | 58 | 81 z | 2 | 2 | 0.125 | 1 | |
| 143 | 2 | 63 | 82 z | 2 | 2 | 0.125 | 1 | |
| 144 | 2 | 64 | 83 z | 2 | 2 | 0.125 | 1 | |
| 145 | 2 | 49 | 50 y | 2 | 4 | 1 | 1 | 0.1 |
| 146 | 2 | 80 | 81 y | 2 | 4 | 1 | 1 | 0.1 |
| 147 | 2 | 82 | 83 v | 2 | 4 | 1 | 1 | 0.1 |
| 148 | 2 | 80 | 82 v | 2 | 4 | 1 | 1 | 0.1 |
| 149 | 2 | 81 | 83 v | 2 | 4 | 1 | 1 | 0.1 |
| 150 | 2 | 49 | 80 v | 2 | 4 | 1 | 1 | 0.1 |
| 151 | 2 | 49 | 81 y | 2 | 4 | 1 | 1 | 0.1 |
| 152 | 2 | 49 | 82 y | 2 | 4 | 1 | 1 | 0.1 |
| 153 | 2 | 49 | 83 y | 2 | 4 | 1 | 1 | 0.1 |
| 154 | 2 | 50 | 80 y | 2 | 4 | 1 | 1 | 0.1 |
| 155 | 2 | 50 | 81 y | 2 | 4 | 1 | 1 | 0.1 |
| 156 | 2 | 50 | 82 v | 2 | 4 | 1 | 1 | 0.1 |
| 157 | 2 | 50 | 83 v | 2 | 4 | 1 | 1 | 0.1 |
| 158 | 2 | 23 | 33 x | 2 | 2 | 0.125 | 3 | |
| 159 | 2 | 24 | 34 x | 2 | 2 | 0.125 | 3 | |
| 160 | 2 | 33 | 37 x | 2 | 2 | 0.125 | 3 | |
| 161 | 2 | 34 | 38 x | 2 | 2 | 0.125 | 3 | |
| 162 | 2 | 37 | 47 x | 2 | 2 | 0.125 | 3 | |
| 163 | 2 | 38 | 48 x | 2 | 2 | 0.125 | 3 | |
| 164 | 2 | 23 | 34 x | 2 | 2 | 0.125 | 3 | |
| 165 | 2 | 24 | 33 x | 2 | 2 | 0.125 | 3 | |
| 166 | 2 | 33 | 38 x | 2 | 2 | 0.125 | 3 | |
| 167 | 2 | 34 | 37 x | 2 | 2 | 0.125 | 3 | |
| 168 | 2 | 37 | 48 x | 2 | 2 | 0.125 | 3 | |
| 169 | 2 | 38 | 47 x | 2 | 2 | 0.125 | 3 | |

| | | | | | | | | |
|-----|---|-----|-------|---|---|-------|---|-----|
| 170 | 2 | 23 | 37 x | 2 | 2 | 0.125 | 3 | |
| 171 | 2 | 24 | 38 x | 2 | 2 | 0.125 | 3 | |
| 172 | 2 | 13 | 84 v | 2 | 2 | 0.25 | 1 | |
| 173 | 2 | 14 | 85 v | 2 | 2 | 0.25 | 1 | |
| 174 | 2 | 13 | 86 y | 2 | 2 | 0.25 | 1 | |
| 175 | 2 | 14 | 87 y | 2 | 2 | 0.25 | 1 | |
| 176 | 2 | 69 | 88 y | 2 | 2 | 0.25 | 1 | |
| 177 | 2 | 70 | 89 y | 2 | 2 | 0.25 | 1 | |
| 178 | 2 | 69 | 90 y | 2 | 2 | 0.25 | 1 | |
| 179 | 2 | 70 | 91 v | 2 | 2 | 0.25 | 1 | |
| 180 | 2 | 128 | 49 v | 2 | 4 | 1 | 1 | 0.1 |
| 181 | 2 | 128 | 50 v | 2 | 4 | 1 | 1 | 0.1 |
| 182 | 2 | 128 | 80 y | 2 | 4 | 1 | 1 | 0.1 |
| 183 | 2 | 128 | 81 y | 2 | 4 | 1 | 1 | 0.1 |
| 184 | 2 | 128 | 82 y | 2 | 4 | 1 | 1 | 0.1 |
| 185 | 2 | 128 | 83 y | 2 | 4 | 1 | 1 | 0.1 |
| 186 | 2 | 112 | 114 z | 2 | 4 | 1 | 1 | 0.1 |
| 187 | 2 | 113 | 115 z | 2 | 4 | 1 | 1 | 0.1 |
| 188 | 2 | 114 | 116 z | 2 | 4 | 1 | 1 | 0.1 |
| 189 | 2 | 115 | 117 z | 2 | 4 | 1 | 1 | 0.1 |
| 190 | 2 | 102 | 104 z | 2 | 4 | 1 | 1 | 0.1 |
| 191 | 2 | 103 | 105 z | 2 | 4 | 1 | 1 | 0.1 |
| 192 | 2 | 104 | 112 z | 2 | 4 | 1 | 1 | 0.1 |
| 193 | 2 | 105 | 113 z | 2 | 4 | 1 | 1 | 0.1 |
| 194 | 2 | 104 | 114 z | 2 | 4 | 1 | 1 | 0.1 |
| 195 | 2 | 105 | 115 z | 2 | 4 | 1 | 1 | 0.1 |
| 196 | 2 | 104 | 116 z | 2 | 4 | 1 | 1 | 0.1 |
| 197 | 2 | 105 | 117 z | 2 | 4 | 1 | 1 | 0.1 |
| 198 | 2 | 100 | 102 z | 2 | 4 | 1 | 1 | 0.1 |
| 199 | 2 | 101 | 103 z | 2 | 4 | 1 | 1 | 0.1 |
| 200 | 2 | 100 | 112 z | 2 | 4 | 1 | 1 | 0.1 |
| 201 | 2 | 101 | 113 z | 2 | 4 | 1 | 1 | 0.1 |
| 202 | 2 | 100 | 114 z | 2 | 4 | 1 | 1 | 0.1 |
| 203 | 2 | 101 | 115 z | 2 | 4 | 1 | 1 | 0.1 |
| 204 | 2 | 100 | 116 z | 2 | 4 | 1 | 1 | 0.1 |
| 205 | 2 | 101 | 117 z | 2 | 4 | 1 | 1 | 0.1 |
| 206 | 2 | 112 | 118 z | 2 | 4 | 1 | 1 | 0.1 |
| 207 | 2 | 113 | 119 z | 2 | 4 | 1 | 1 | 0.1 |
| 208 | 2 | 114 | 118 z | 2 | 4 | 1 | 1 | 0.1 |
| 209 | 2 | 115 | 119 z | 2 | 4 | 1 | 1 | 0.1 |
| 210 | 2 | 116 | 118 z | 2 | 4 | 1 | 1 | 0.1 |
| 211 | 2 | 117 | 119 z | 2 | 4 | 1 | 1 | 0.1 |
| 212 | 2 | 102 | 112 z | 2 | 4 | 1 | 1 | 0.1 |
| 213 | 2 | 103 | 113 z | 2 | 4 | 1 | 1 | 0.1 |
| 214 | 2 | 102 | 114 z | 2 | 4 | 1 | 1 | 0.1 |
| 215 | 2 | 103 | 115 z | 2 | 4 | 1 | 1 | 0.1 |
| 216 | 2 | 102 | 116 z | 2 | 4 | 1 | 1 | 0.1 |
| 217 | 2 | 103 | 117 z | 2 | 4 | 1 | 1 | 0.1 |
| 218 | 2 | 120 | 122 z | 2 | 4 | 1 | 1 | 0.1 |
| 219 | 2 | 121 | 123 z | 2 | 4 | 1 | 1 | 0.1 |
| 220 | 2 | 122 | 124 z | 2 | 4 | 1 | 1 | 0.1 |
| 221 | 2 | 123 | 125 z | 2 | 4 | 1 | 1 | 0.1 |
| 222 | 2 | 108 | 110 z | 2 | 4 | 1 | 1 | 0.1 |
| 223 | 2 | 109 | 111 z | 2 | 4 | 1 | 1 | 0.1 |
| 224 | 2 | 110 | 120 z | 2 | 4 | 1 | 1 | 0.1 |
| 225 | 2 | 111 | 121 z | 2 | 4 | 1 | 1 | 0.1 |
| 226 | 2 | 110 | 122 z | 2 | 4 | 1 | 1 | 0.1 |
| 227 | 2 | 111 | 123 z | 2 | 4 | 1 | 1 | 0.1 |
| 228 | 2 | 110 | 124 z | 2 | 4 | 1 | 1 | 0.1 |
| 229 | 2 | 111 | 125 z | 2 | 4 | 1 | 1 | 0.1 |
| 230 | 2 | 106 | 108 z | 2 | 4 | 1 | 1 | 0.1 |
| 231 | 2 | 107 | 109 z | 2 | 4 | 1 | 1 | 0.1 |
| 232 | 2 | 106 | 120 z | 2 | 4 | 1 | 1 | 0.1 |
| 233 | 2 | 107 | 121 z | 2 | 4 | 1 | 1 | 0.1 |
| 234 | 2 | 106 | 122 z | 2 | 4 | 1 | 1 | 0.1 |

| | | | | | | | | |
|-----|---|-----|-------|---|---|---|---|-----|
| 235 | 2 | 107 | 123 z | 2 | 4 | 1 | 1 | 0.1 |
| 236 | 2 | 106 | 124 z | 2 | 4 | 1 | 1 | 0.1 |
| 237 | 2 | 107 | 125 z | 2 | 4 | 1 | 1 | 0.1 |
| 238 | 2 | 126 | 120 z | 2 | 4 | 1 | 1 | 0.1 |
| 239 | 2 | 127 | 121 z | 2 | 4 | 1 | 1 | 0.1 |
| 240 | 2 | 126 | 122 z | 2 | 4 | 1 | 1 | 0.1 |
| 241 | 2 | 127 | 123 z | 2 | 4 | 1 | 1 | 0.1 |
| 242 | 2 | 126 | 124 z | 2 | 4 | 1 | 1 | 0.1 |
| 243 | 2 | 127 | 125 z | 2 | 4 | 1 | 1 | 0.1 |
| 244 | 2 | 108 | 120 z | 2 | 4 | 1 | 1 | 0.1 |
| 245 | 2 | 109 | 121 z | 2 | 4 | 1 | 1 | 0.1 |
| 246 | 2 | 108 | 122 z | 2 | 4 | 1 | 1 | 0.1 |
| 247 | 2 | 109 | 123 z | 2 | 4 | 1 | 1 | 0.1 |
| 248 | 2 | 108 | 124 z | 2 | 4 | 1 | 1 | 0.1 |
| 249 | 2 | 109 | 125 z | 2 | 4 | 1 | 1 | 0.1 |

| NUMBER | NAME | E | v | YIELD | DENSITY |
|--------|-----------|----------|---|-----------------|---------|
| 1 | CroMoly | 30000000 | | 0.292 62500 | 0.282 |
| 2 | Rigid | 30000000 | | 0.292 999999999 | 0 |
| 3 | FlexStand | 2000000 | | 0.425 999999999 | 0 |

| 5 | | | | | | | | |
|--------|------|-------|-------|-------|-------|-------|-------|---|
| NUMBER | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM | |
| 1 | 118 | 0 | 110 | 0 | 0 | 0 | 0 | 0 |
| 2 | 119 | 0 | 110 | 0 | 0 | 0 | 0 | 0 |
| 3 | 122 | 0 | 0 | 413 | 0 | 0 | 0 | 0 |
| 4 | 123 | 0 | 0 | 413 | 0 | 0 | 0 | 0 |
| 5 | 126 | 0 | 590 | 0 | 0 | 0 | 0 | 0 |
| 6 | 127 | 0 | 590 | 0 | 0 | 0 | 0 | 0 |
| 7 | 128 | 0 | -410 | 320 | -1040 | 0 | 0 | 0 |

| NUMBER | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM | |
|--------|------|-------|-------|-------|-------|-------|-------|---|
| 1 | 114 | 0 | 0 | -393 | 3930 | 0 | 0 | 0 |
| 2 | 115 | 0 | 0 | -393 | 3930 | 0 | 0 | 0 |
| 3 | 118 | 0 | 562 | 0 | 0 | 0 | 0 | 0 |
| 4 | 119 | 0 | 562 | 0 | 0 | 0 | 0 | 0 |
| 5 | 122 | 0 | 0 | -96 | 960 | 0 | 0 | 0 |
| 6 | 123 | 0 | 0 | -96 | 960 | 0 | 0 | 0 |
| 7 | 126 | 0 | 138 | 0 | 0 | 0 | 0 | 0 |
| 8 | 127 | 0 | 138 | 0 | 0 | 0 | 0 | 0 |

| NUMBER | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM | |
|--------|------|-------|-------|-------|-------|-------|-------|---|
| 1 | 118 | 436 | 624 | 0 | 0 | 0 | 0 | 0 |
| 2 | 119 | 34 | 48 | 0 | 0 | 0 | 0 | 0 |
| 3 | 126 | 473 | 676 | 0 | 0 | 0 | 0 | 0 |
| 4 | 127 | 36 | 52 | 0 | 0 | 0 | 0 | 0 |

| NUMBER | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM | |
|--------|------|-------|-------|-------|-------|-------|-------|---|
| 1 | 118 | 190 | 380 | 0 | 0 | 0 | 0 | 0 |
| 2 | 119 | 0 | 31 | 0 | 0 | 0 | 0 | 0 |
| 3 | 122 | 0 | 0 | 374 | 0 | 0 | 0 | 0 |
| 4 | 123 | 0 | 0 | 151 | 0 | 0 | 0 | 0 |
| 5 | 126 | 374 | 748 | 0 | 0 | 0 | 0 | 0 |
| 6 | 127 | 151 | 303 | 0 | 0 | 0 | 0 | 0 |
| 7 | 128 | 0 | -410 | 320 | -1040 | 0 | 0 | 0 |

| NUMBER | NODE | X-MAG | Y-MAG | Z-MAG | X-MOM | Y-MOM | Z-MOM | |
|--------|------|-------|-------|-------|-------|-------|-------|---|
| 1 | 114 | 0 | 0 | -351 | 3510 | 0 | 0 | 0 |
| 2 | 115 | 0 | 0 | -146 | 1460 | 0 | 0 | 0 |
| 3 | 118 | 351 | 703 | 0 | 0 | 0 | 0 | 0 |
| 4 | 119 | 146 | 291 | 0 | 0 | 0 | 0 | 0 |
| 5 | 122 | 0 | 0 | -213 | 2130 | 0 | 0 | 0 |
| 6 | 123 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 7 | 126 | 213 | 425 | 0 | 0 | 0 | 0 | 0 |
| 8 | 127 | 0 | 20 | 0 | 0 | 0 | 0 | 0 |
